Bunched Beam Stochastic Cooling and Coherent Lines

M. Blaskiewicz, J.M. Brennan C-AD BNL

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- Coherence in Proton Beams

Stochastic cooling considered for SPS, and Tevatron (80s).
Unexpected RF activity swamps the Schottky signal (85s).
Heavy Ion cooling in RHIC under construction (now)

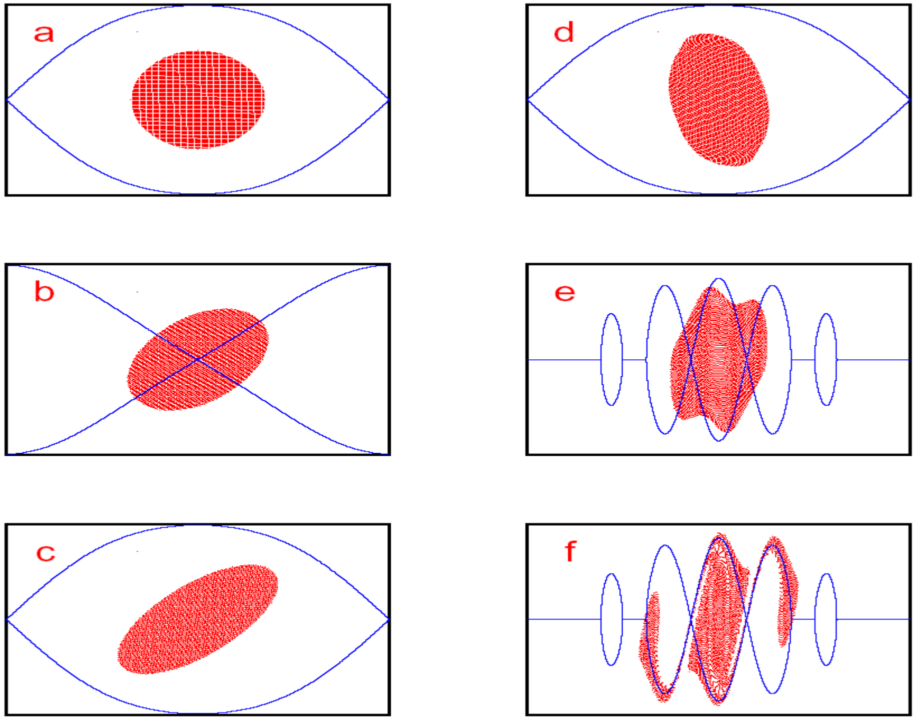
Coherence in Heavy Ion Beams

Two distinct types:

2) Strong revolution lines
3) Strong signals associated with synchrotron motion

We see the first type with heavy ions and both with protons.

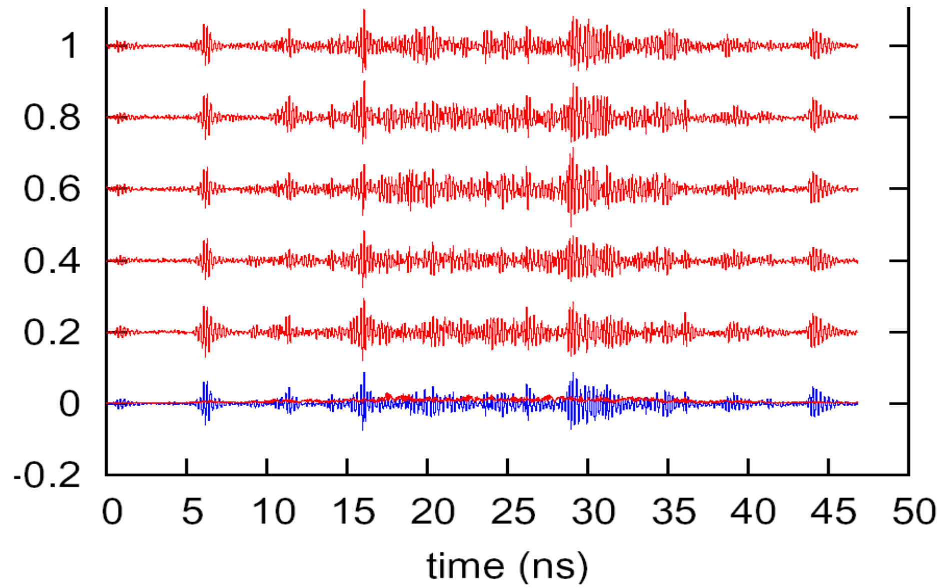
Heavy ions are "rebucketed" to shorten the bunch and combat IBS



Time domain signals
from different Cu bunches

$$S_1(t) = \sum_{k=0}^{3} S_0(t - k \times 5ns)$$

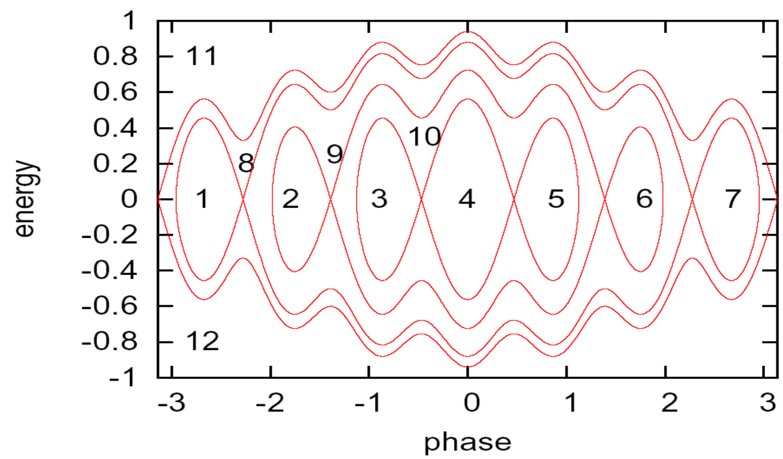
Blue is average, which is filtered out. Orange is rms Schottky. Thanks Agilent!



Separatricies for
Storage bucket

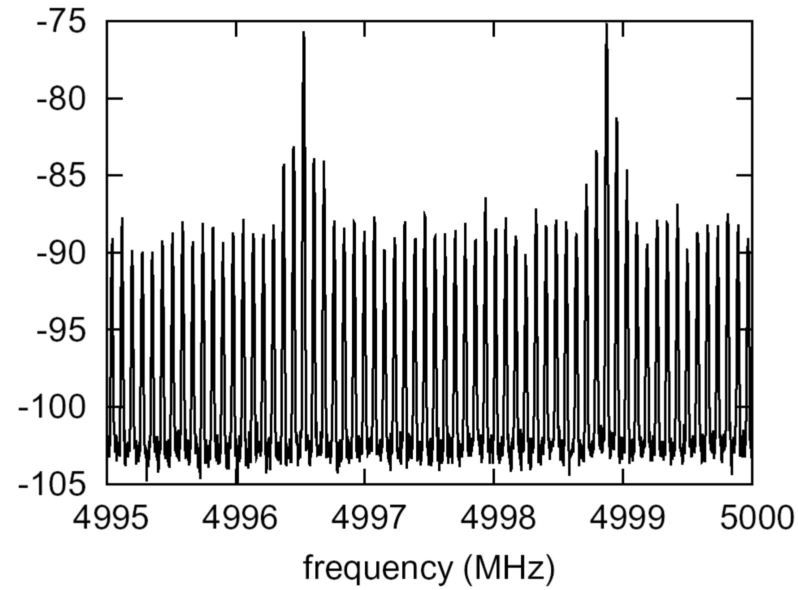
$$P_k(H) = \int_{regionk} dp d\phi \Psi(p,\phi,t)\delta[H - p^2/2 - U(\phi)]. \qquad (1)$$

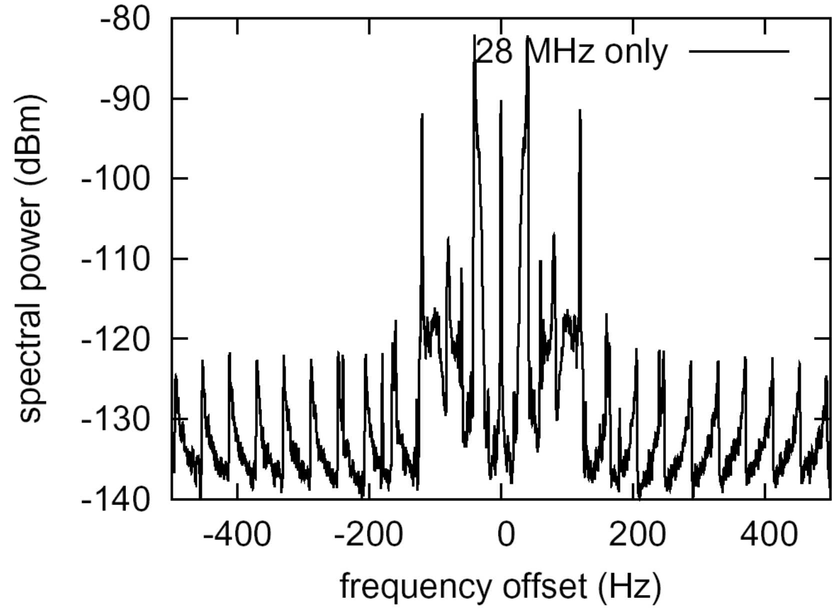
$$\Psi_k(p,\phi) = P_k[H(p,\phi)]/T_k[H(p,\phi)]$$



Coherence in Proton Beams

Wide band (top) observed with
28 bunches in 30 bunch fill pattern.
h=360. Similar to what would be
expected due to hard edge bunch.
Zoom of the strong right peak (bottom).
90% of the power in dipole lines (40 Hz)
10% in rev and +/- 120 Hz.
<1% Schottky.





RF phase noise?

Stroboscopic average of phase space (top) via

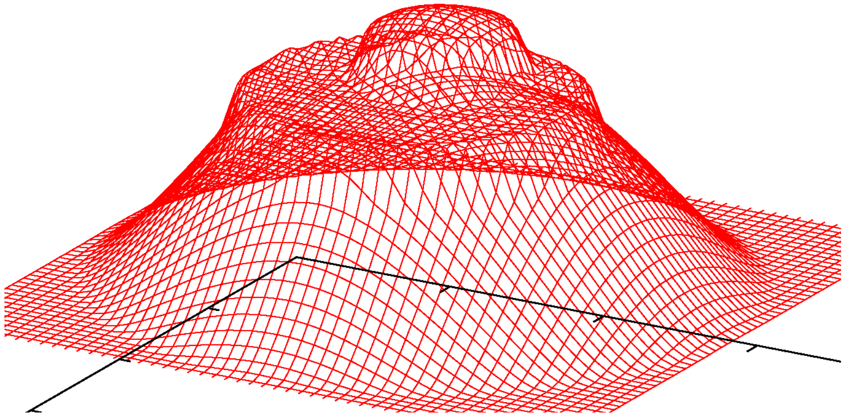
$$\frac{d^2\varphi}{dt^2} + \omega_{s0}^2 \sin[\varphi - \varphi_0(t)] = 0$$

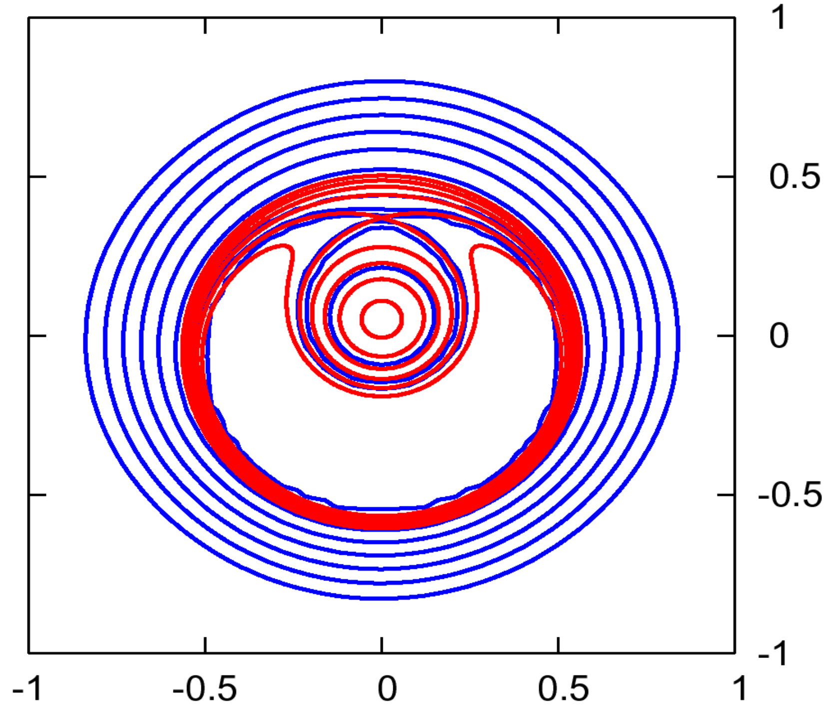
$$\varphi_0(t) = \alpha(t)\sin(\nu\omega_{s0}t) \le 0.001$$

Contour plot (bottom) of phase space (blue) and 1st order perturbative Hamiltonian (red)

$$H(\chi, J) = H_0(J) - \nu J - \frac{\alpha\nu^2}{2}\sqrt{2J}\cos\chi$$

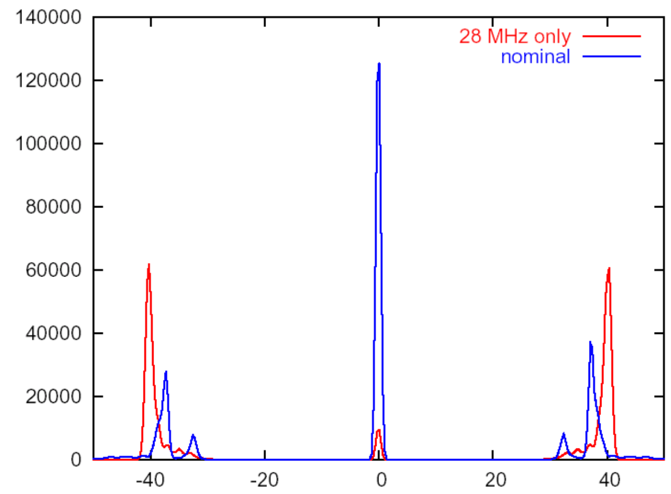
Discontinuous derivatives like $\sqrt{|t|}$ in line density





Actual driving term is not monochromatic

Varying synchrotron frequency via
RF voltage changes observed spectra.
Need to allow for stochastic driving term



$$\phi_0(t) = \int_{\infty}^{\infty} d\Omega \alpha(\Omega) \exp[(\varepsilon - i\Omega)t]$$

$$< \alpha(\Omega_1)\alpha^*(\Omega_2) >= P_\alpha(\Omega_1)\delta(\Omega_1 - \Omega_2)$$

1st order perturbation theory on Vlasov eq

$$f(\psi, J, t) = f_0(J) + f_1$$

$$f_1(\psi, J, t) = \int_{\infty}^{\infty} d\Omega \sum_{m=\pm 1} \frac{\exp[im\psi + (\varepsilon - i\Omega)t]}{\varepsilon - i\Omega + im\omega_s(J)} \Omega^2 \alpha(\Omega) \sqrt{J/2\omega_{s0}} \frac{df_0}{dJ}$$

$$< |\tilde{I}(\omega)|^2 >= \sum_{k=-\infty}^{\infty} |I_k|^2 \frac{P_\alpha(\omega - k\omega_0)}{4\omega_{s0}^2} \left| \int dJ \frac{df_0}{dJ} \sum_{m=\pm 1} \frac{m(\omega - k\omega_0)^2 \hat{\phi} \mathrm{J}_1(k\hat{\phi}/h)}{\varepsilon - i(\omega - k\omega_0) + im\omega_s(J)} \right|^2$$

Bunched versus Coasting Beams

Compare via BTF

Frequency of a given particle $\omega = \omega_0 + d\phi/dt$

Voltage due to kicker $V_K(t) = V_K \exp(-i\tilde{\omega}t + \epsilon t)$

Effect on a particle $\left.\frac{d\omega}{dt}\right|_K = \frac{-\eta\omega_0^2}{\beta^2 E_T/q}\delta_p(\theta - \theta_K)\, V_K(t)$

1st order Vlasov perturbation
Standard tricks

$$\frac{\partial\Psi_1}{\partial t} + \omega_s(J)\frac{\partial\Psi_1}{\partial\psi} = -\left.\frac{d\omega}{dt}\right|_K \sqrt{\frac{2J}{\omega_{s0}}}\cos\psi\frac{d\Psi_0}{dJ}$$

$$I(\theta_P, \tilde{\omega}) = C\sum_m \frac{e^{im(\theta_P - \theta_K)}}{m}\int dJ\frac{d\Psi_0}{dJ}\sum_n J_n^2(m\sqrt{2J/\omega_{s0}})\frac{n}{\epsilon - i(\tilde{\omega} - m\omega_0) + in\omega_s(J)}$$

General features look good

Generic features are well modeled by a coasting beam transfer function.