Cooling of Ions and $\overline{\mathrm{p}}$ with Magnetized Electrons*

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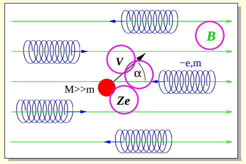
Energy loss of ions by collisions with magnetized electrons

- Goal: $\frac{dE}{dt} = \vec{F}(\vec{V}, Z, n_e, T_e, B) \cdot \vec{V} = \boxed{\frac{dE}{ds}} |\vec{V}| \quad \rightarrow \quad \frac{d}{dt} T_i(t)$
- Challenges:
  - Two-body problem is chaotic
  - High charge states of ions
  - Strong magnetic field
  - Electron-electron-interaction (collective effects)



▶ Requires different, complementary theoretical approaches:

- Analytical: perturbative treatment of binary collisions (BC)[6,8,10] and dielectric linear response (LR)[4,5,8,11]
- Numerical simulations: Classical Trajectory Monte Carlo (CTMC)[2,3,7] and Particle-In-Cell/Test-Particle (PIC) [1,5,10,11]

Binary collision model

- The motion of the center-of-mass (cm) and the relative motion do not separate.
- But for an ion mass M much larger than the electron mass m it is approximately:

☐ cm-motion: $\vec{v}_i = \text{const} + O\left(\frac{m}{M}\right), \qquad \vec{V}_{cm} = \vec{v}_i + O\left(\frac{m}{M}\right)$

☐ relative motion: $\frac{d}{dt} m\vec{v}_r = -\nabla\Phi(r_r) - e\,(\vec{v}_r \times \vec{B}) - e\,(\vec{v}_i \times \vec{B}) + O\left(\frac{m}{M}\right)$

☐ cm-energy: $\frac{dE_{cm}}{dt} = \vec{V}_{cm} \cdot \frac{d}{dt} M\vec{V}_{cm} = -e\,\vec{v}_i \cdot (\vec{v}_r \times \vec{B}) + O\left(\frac{m}{M}\right)$

☐ integral of motion: $K = \frac{m}{2} v_r^2 + \Phi(r_r) + e\,(\vec{v}_i \times \vec{B}) \cdot \vec{r}_r + O\left(\frac{m}{M}\right)$

- In the binary collision model, the central physical observables are the velocity transfer $\Delta\vec{v}_r$ and the related energy transfer in a single collision

$$\Delta E_i \approx M\vec{v}_i \cdot \Delta\vec{v}_i = -m\vec{v}_i \cdot \Delta\vec{v}_r + M\vec{v}_i \cdot \Delta\vec{v}_{\mathrm{cm}} = -m\vec{v}_i \cdot \Delta\vec{v}_r - \frac{m}{2}\left((\vec{v}_r + \Delta\vec{v}_r)^2 - \vec{v}_r^2\right)$$

- The velocity and energy transfers, $\langle\Delta\vec{v}_r\rangle(\vec{v}_r, \vec{v}_i)$ and $\langle\Delta E_i\rangle(\vec{v}_r, \vec{v}_i)$, for a monochromatic beam of electrons results after integration with respect to the impact parameter b and the initial phase φ of the electron helix.
- Folding with the electron velocity distribution $f(\vec{v}_e)$ yields the drag force $\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}$ and the energy loss $dE_i/ds = \vec{F} \cdot \vec{v}_i / v_i$

Second-order perturbation treatment [6]

- The velocity transfer is calculated up to second order $O(Z^2)$ in the electron-ion interaction from the equations of motion for:
  - ∗ the unperturbed helical motion of the electrons

$$m\dot{\vec{v}}_0 + e\,(\vec{v}_0 \times \vec{B}) = -e\,(\vec{v}_i \times \vec{B}), \qquad \dot{\vec{r}}_0 = \vec{v}_0$$

  - ∗ the first order contribution

$$m\dot{\vec{v}}_1 + e\,(\vec{v}_1 \times \vec{B}) = -\nabla\Phi(r_0), \qquad \dot{\vec{r}}_1 = \vec{v}_1$$

  - ∗ the second order correction

$$m\dot{\vec{v}}_2 + e\,(\vec{v}_2 \times \vec{B}) = -\left[\nabla\Phi(|\vec{r}_0 + \vec{r}_1|) - \nabla\Phi(r_0)\right]$$

  where $\vec{v}_r = \vec{v}_0 + \vec{v}_1 + \vec{v}_2$ and $\vec{r} = \vec{r}_0 + \vec{r}_1 + \vec{r}_2$

- Three regimes, characterzied by Rutherford trajectories, stretched helices and tight helices, are identified where closed expressions for the velocity- and energy transfer can be derived. Interpolating between these regimes yields the final energy transfer.
- The averaging over the impact parameter (or distance of closest approach) b and the initial phase φ can be performed analytically with an upper cut-off $b_{\max}$ accounting for dynamic screening and a lower cut-off $b_{\min}$ excluding hard collisions.

Numerical treatment: Classical Trajectory Monte Carlo [2,7]

- numerical integration of the equations of motion

$$m\dot{\vec{v}}_r = -\nabla\left[-\frac{Ze^2}{r}\exp(-\frac{r}{\lambda})\right] - e\,(\vec{v}_r \times \vec{B}) - e\,(\vec{v}_i \times \vec{B}), \qquad \dot{\vec{r}} = \vec{v}_r$$

through the interaction region ($\sim \lambda$) for given initial values $\vec{v}_r, \vec{v}_i, \vec{b}, \varphi$

⇒ velocity transfer $\Delta\vec{v}_r(\vec{v}_r, \vec{v}_i, \vec{b}, \varphi)$, energy transfer $\Delta E_i(\vec{v}_r, \vec{v}_i, \vec{b}, \varphi)$

- Monte Carlo sampling over $\vec{b}, \varphi$ with $\approx 5 \times 10^4 \ldots 4 \times 10^5$ trajectories per $\vec{v}_r, \vec{v}_i$

$$\Rightarrow \langle \Delta E_i(\vec{v}_r, \vec{v}_i) \rangle = \int d^2b \int_0^{2\pi} \frac{d\varphi}{2\pi} \Delta E_i(\vec{v}_r, \vec{v}_i, \vec{b}, \varphi), \quad \langle \Delta\vec{v}_r(\vec{v}_r, \vec{v}_i) \rangle = \ldots$$

Electron Cooling in Storage Rings

Parameters (TSR, Heidelberg)

Ion	:	C^{6+} $Z = 6$
B	:	0.01 . . . 0.05 T
n_e	:	8×10^6 cm^{-3}
$k_B T_\perp$	:	11.5 meV
$k_B T_\parallel$	:	0.1 meV
$T_\perp / T_\parallel$	:	115
$v_{th\perp} = \left(\frac{k_B T_\perp}{m}\right)^{1/2}$	:	4.5×10^4 m/s
$v_{th\parallel} = \left(\frac{k_B T_\parallel}{m}\right)^{1/2}$	:	4.2×10^3 m/s
$v_{th\perp} / v_{th,\parallel}$	:	10.7
$\lambda_{D\perp} = \left(\frac{\epsilon_0 k_B T_\perp}{e^2 n}\right)^{1/2}$	:	2.8×10^{-4} m
$\lambda_{D\parallel} = \left(\frac{\epsilon_0 k_B T_\parallel}{e^2 n}\right)^{1/2}$	:	2.6×10^{-5} m
$b_{\max} = \lambda$	:	1.0×10^{-4} m

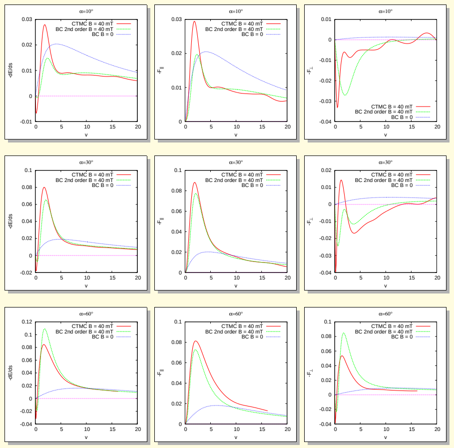
Scales

$$v_i / v_{\mathrm{th}\parallel} \quad \rightarrow \quad v,\ v_i$$

$$\frac{dE/ds}{\left(\frac{Z^2 e^4 n_e}{4\pi\epsilon_0^2 m v_{\mathrm{th}\parallel}^2}\right)} \rightarrow dE/ds$$

with $\frac{Z^2 e^4 n_e}{4\pi\epsilon_0^2 m v_{\mathrm{th}\parallel}^2} = 0.18\ \frac{\text{eV}}{\text{cm}}$

■ **CTMC versus 2nd order BC for different α and B = 40 mT**

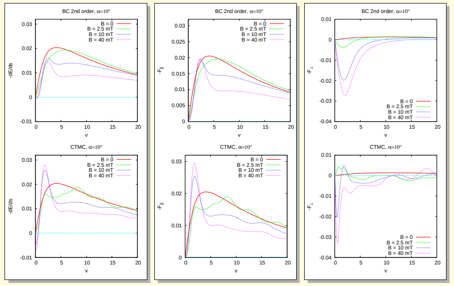


▶ Very good qualitative agreement and a quantitative agreement within less than a factor 2 between the perturbation treatment (2nd order BC) and the numerical simulations (CTMC).

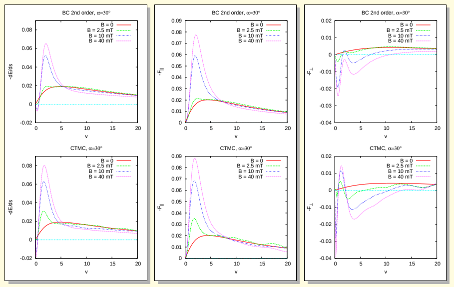
References

[1] G. Zwicknagel, C. Toepffer and P.-G. Reinhard, Physics Reports **309** (1999) 117.
[2] G. Zwicknagel, in *Non-neutral Plasma Physics III*, eds. J.J. Bollinger, R.L. Spencer and R.C. Davidson, AIP Conference Proceedings **498**, 1999, p 469.
[3] G. Zwicknagel, Nucl.Instr.Meth. in Phys.Res. A **441** (2000) 44.
[4] H. B. Nersisyan, M. Walter, G. Zwicknagel, Phys.Rev.E **61** (2000) 7022.
[5] M. Walter, C. Toepffer, G. Zwicknagel, Nucl.Instr.Meth. in Phys.Res. B **168** (2000) 347.
[6] C. Toepffer, Phys.Rev.A **66** (2002) 022714.
[7] G. Zwicknagel and C. Toepffer, in *Non-neutral Plasma Physics IV*, eds. F. Anderegg, L. Schweikhard and C.F. Driscoll, AIP Conference Proceedings **606**, 2002, p 499.
[8] H. B. Nersisyan, G. Zwicknagel, and C. Toepffer, Phys.Rev.E **67** (2003) 026411.
[9] B. Möllers, C. Toepffer, M. Walter, and G. Zwicknagel, Nucl.Instr.Meth. in Phys.Res. B **205** (2003) 285.
[10] B. Möllers, M. Walter, G. Zwicknagel, C. Carli and C. Toepffer, Nucl.Instr.Meth. in Phys.Res. B **207** (2003) 462.
[11] M. Walter, C. Toepffer, G. Zwicknagel, Eur.Phys.J. D **35** (2005) 527.

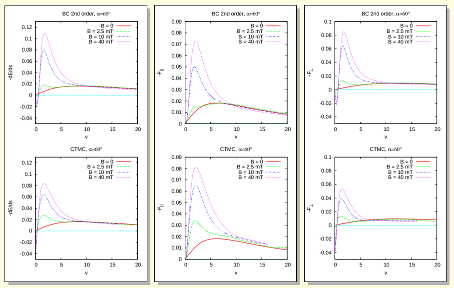
■ **Energy loss and drag forces for different B and $\alpha = 10^o$**



■ **Energy loss and drag forces for different B and $\alpha = 30^o$**

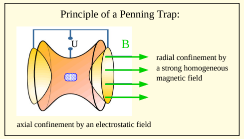


■ **Energy loss and drag forces for different B and $\alpha = 60^o$**



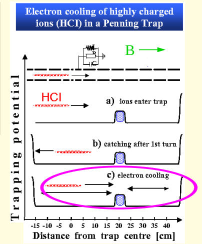
- 'Anti-cooling' for small v
- 'Anti-friction' in perpendicular direction for small $v, v_\perp$

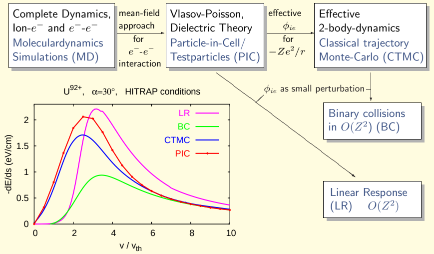
Electron Cooling in Penning Traps



Typical parameters:

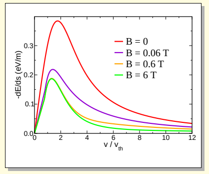
- $Z = -1, 1 \ldots 92$
- $B \lesssim 6$ T
- $n_e \approx 10^7$ cm^{-3}
- $T_e \approx 4$ K



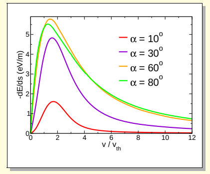


Energy loss in isotropic magnetized electrons (from CTMC)

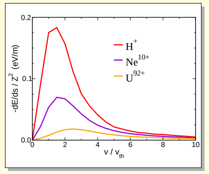
H^+; $\alpha = 30^o$, $n_e = 10^7$cm^{-3}, $T_e = 4$ K



Ne^{10+}; $B = 6$T, $n_e = 10^7$cm^{-3}, $T_e = 4$ K



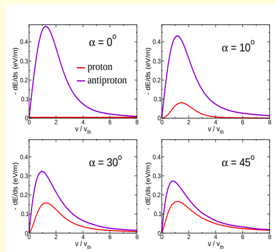
$\alpha = 30^o, B = 6$T$, n_e = 10^7$cm$^{-3}, T_e = 4$ K



■ Essential features (summary):

- Reduction of dE/ds with increasing B
- $dE/ds(\vec{V}), \vec{F}(\vec{V})$ highly anisotropic
- Z-scaling $Z^x, x < 2$ $(Z > 0)$
- $dE/ds(-|Z|) > dE/ds(|Z|)$

■ **Protons p versus antiprotons $\overline{\mathrm{p}}$** (B=6 T, $n_e = 10^7$ cm^{-3}, $T_e = 4$ K)



- At large magnetic fields and $V \lesssim v_{th}$:

$$\boxed{dE/ds(\overline{p}) > dE/ds(p)}$$