Transverse-Longitudinal Correlations: FEL Performance and Emittance Exchange

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COOL05

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1. Introduction

- The energy and longitudinal position of particles are often correlated to good advantage. This is done for bunch compression and in the final focus region of linear colliders.
- Here we consider two special types of correlations.
- The first is energy correlated with transverse amplitude which is the idea behind a “beam conditioner”.
- The second is correlations needed for emittance transfer from transverse to longitudinal phase space.
- Either one of these very much improve the performance of an FEL.

2. References

•A. M. Sessler, D. H. Whittum, L-H. Yu, “RF Beam Conditioning for the FEL”, February 1991 (unpublished).

•A.M. Sessler, D.W. Whittum, and L.-H. Yu, " Radio-frequency beam conditioner for fast wave free electron generators of coherent radiation", Physical Review Letters **66**, 309 (1992).

•N. A.Vinokurov, Nucl. Inst.and Meth. **A 375**, 264 (1996)

•P. Emma and G. Stupakov, “Limitations of Electron Beam Conditioning for Free-Electron Lasers”, PRST AB **6**, 030701 (2003)

2. References (Cont)

•A. Wolski et al, "Beam Conditioning for Free Electron Lasers: Consequences and Methods", PRST AB **7,** 080701 (2004).

•P. Emma and G. Stupakov, "Controlling Emittance Growth in an FEL Bearm Conditioner", EPAC 2004, p.503.

•C. B. Schroeder et al, " Electron Beam Conditioning by Compton Backscattering", PRL **93**, 194801 (2004).

•A.A. Zholents, "Laser Assisted Electron Beam Conditioning forFree electron Lasers", PRST AB **8**, 050701 (2005).

3. Conditioning Concept

- Resonance condition for FEL requires a specific average velocity: after each undulator period, electrons fall behind the laser field by exactly one wavelength.
- The usual resonance condition assumes zero emittance. Adding correlations of transverse amplitude with energy brings more particles into resonance.
- For a zero amplitude particle, the typical angle is K/γ, where K is the normalized strength of the undulator.
- Then $\frac{v_z^2}{c^2} = 1 - \frac{1}{\gamma^2} - \frac{\bar{v}_\perp^2}{c^2} \simeq 1 - \frac{1+K^2}{\gamma^2}$
- Slippage after one undulator period should be λ:

$$\lambda = \Delta z = \left(1 - \frac{v_z}{c}\right)\lambda_w \simeq \frac{1}{2}\left(1 - \frac{v_z^2}{c^2}\right)\lambda_w = \frac{1+K^2}{2\gamma^2}\lambda_w$$

3. Conditioning Concept (Cont)

This is the basic resonance condition. For large γ, the angle and $v_\perp/c$ are roughly the same.

For non-zero emittance, the average angle in terms of the normalized emittances is:

$$\langle \theta_\epsilon^2 \rangle \simeq \frac{2\pi(\epsilon_{Nx} + \epsilon_{Ny})}{\gamma \lambda_\beta}$$

Here $\lambda_\beta = 2\pi\beta_x = 2\pi\beta_y$

The angles from the emittances and undulator are uncorrelated, and add in quadrature.

Modified equation for v_z:

$$\langle \theta_\epsilon^2 \rangle + \frac{v_z^2}{c^2} \simeq 1 - \frac{1+K^2}{\gamma^2}$$

3. Conditioning Concept (Cont)

To have uniform v_z requires an energy shift $\Delta\gamma$ from the zero emittance case to balance out the emittance term:

$$\frac{2\pi(\epsilon_{Nx} + \epsilon_{Ny})}{\gamma\lambda_\beta} \simeq \langle\theta_\epsilon^2\rangle = \frac{1+K^2}{\gamma^2}\frac{2\Delta\gamma}{\gamma}$$

Note $\Delta\gamma/\gamma << 1 \Rightarrow \lambda_\beta >> \pi\gamma(\varepsilon_{Nx} + \varepsilon_{Ny})/(1+K^2)$

Using the resonance condition and taking $\varepsilon_{Nx}=\varepsilon_{Ny}=\varepsilon_N$,

$$\Delta\gamma = \pi\frac{\lambda_w}{\lambda}\frac{\epsilon_N}{\lambda_\beta}$$

If the gain length $\leq \lambda_\beta$, no averaging over betatron oscillation: then what matters is the peak angle, near axis, where the fields are strongest.

This doubles the conditioning required.

4. Applications

Simulations using GENESIS, with energy-amplitude correlations added to the particle loading subroutines. All runs use amplifying mode, with initial seed; obtain gain length and saturation. Numerical work by Gregg Penn.

The conditioning parameter κ is defined as

$$\Delta\gamma = \kappa \times (J_x + J_y),$$

where J_x is the normalized action, with $< J_x > = \varepsilon_{Nx}$:

i

$$J_x \simeq \frac{1}{2}\gamma \left[\frac{2\pi x^2}{\lambda_\beta} + \frac{\lambda_\beta}{2\pi} \left(\frac{v_x}{c} + \frac{2\pi \alpha x}{\lambda_\beta} \right)^2 \right]$$

4. Applications (Cont)

Proper conditioning requires $\kappa \simeq \frac{\pi}{\lambda_\beta} \frac{\lambda_w}{\lambda}$

Interpretation of conditioning parameter:

for $\kappa = 1\ \mu m^{-1}$, a beam with $\varepsilon_{Nx} = \varepsilon_{Ny} = 2\ \mu m$ has $\Delta\gamma = 4$, i.e., an electron at typical amplitude has 2 MeV more energy than at zero amplitude.

Specific examples are given below.
In the following plots, red = nominal case, green = 2 x emittance, black = largest emittance; points = unconditioned, lines = conditioned.

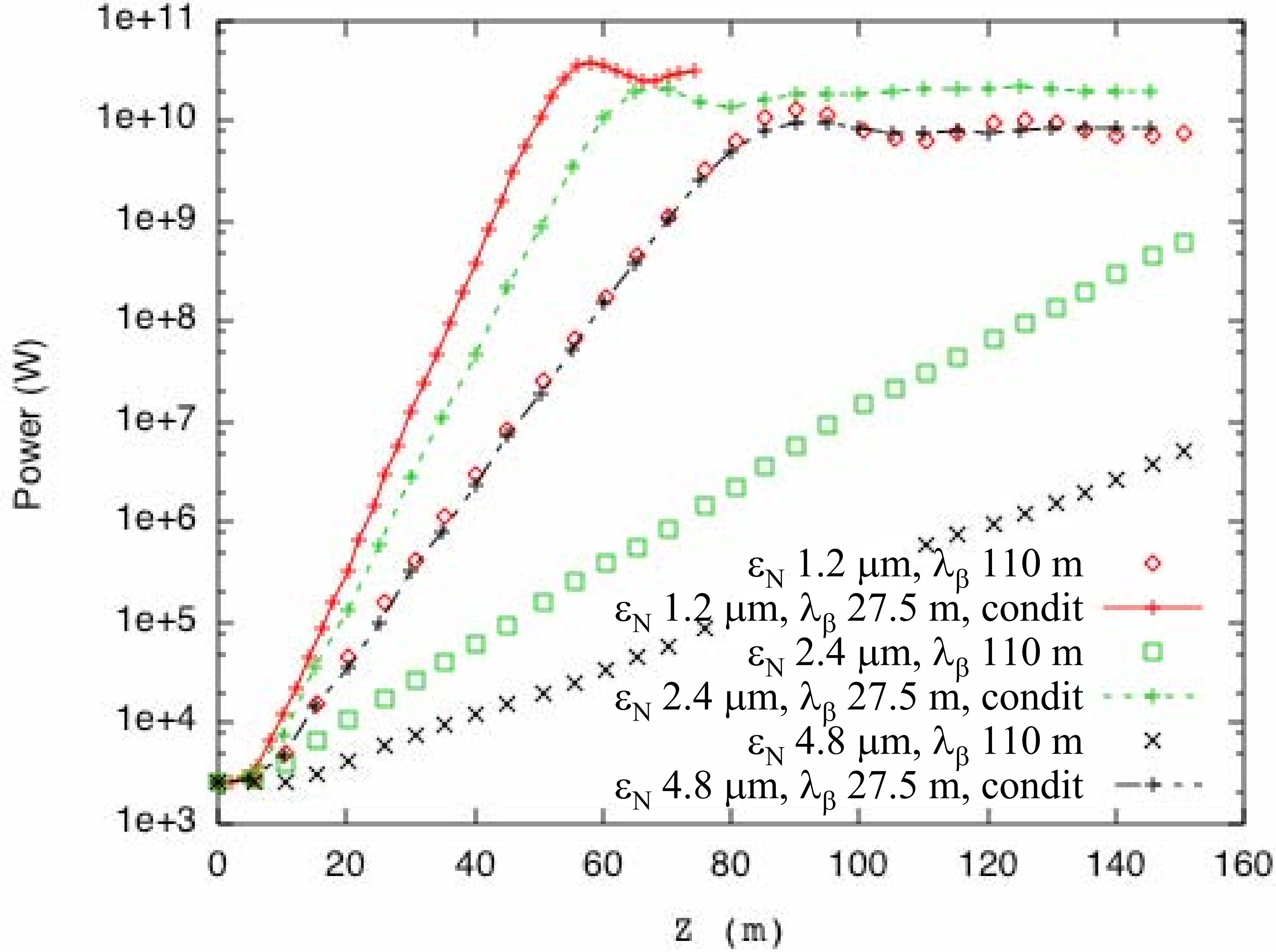
Conditioned beams are optimized at smaller beta functions, leading to further improvements.
Indicated on plots by '+'s overlapping a line.

LCLS

Parameters:

- radiation wavelength 1.5 Å
- 14.3 GeV, $\Delta\gamma/\gamma = 1 \times 10^{-4}$
- 1.2 μm emittance
- peak current 3.4 kA
- undulator: $\lambda_w = 3$ cm, $K = 2.62$
  - $\lambda_\beta \approx 110$ m, matched $\kappa = 5.8\ \mu m^{-1}$

LCLS, vary emittance, optimal λ_β



Greenfield FEL

Possible scheme to achieve highly energetic (30 keV) photons, radiation wavelength 0.4 Å

Low and high energy options.

Both cases have peak current of 3.5 kA.

Nominal emittance 1.2 μm, but consider emittances as low as 0.1 μm.

Nominal $\lambda_\beta \approx 110$ m in both cases.

High energy parameters:

27.8 GeV, $\Delta\gamma/\gamma = 1 \times 10^{-4}$

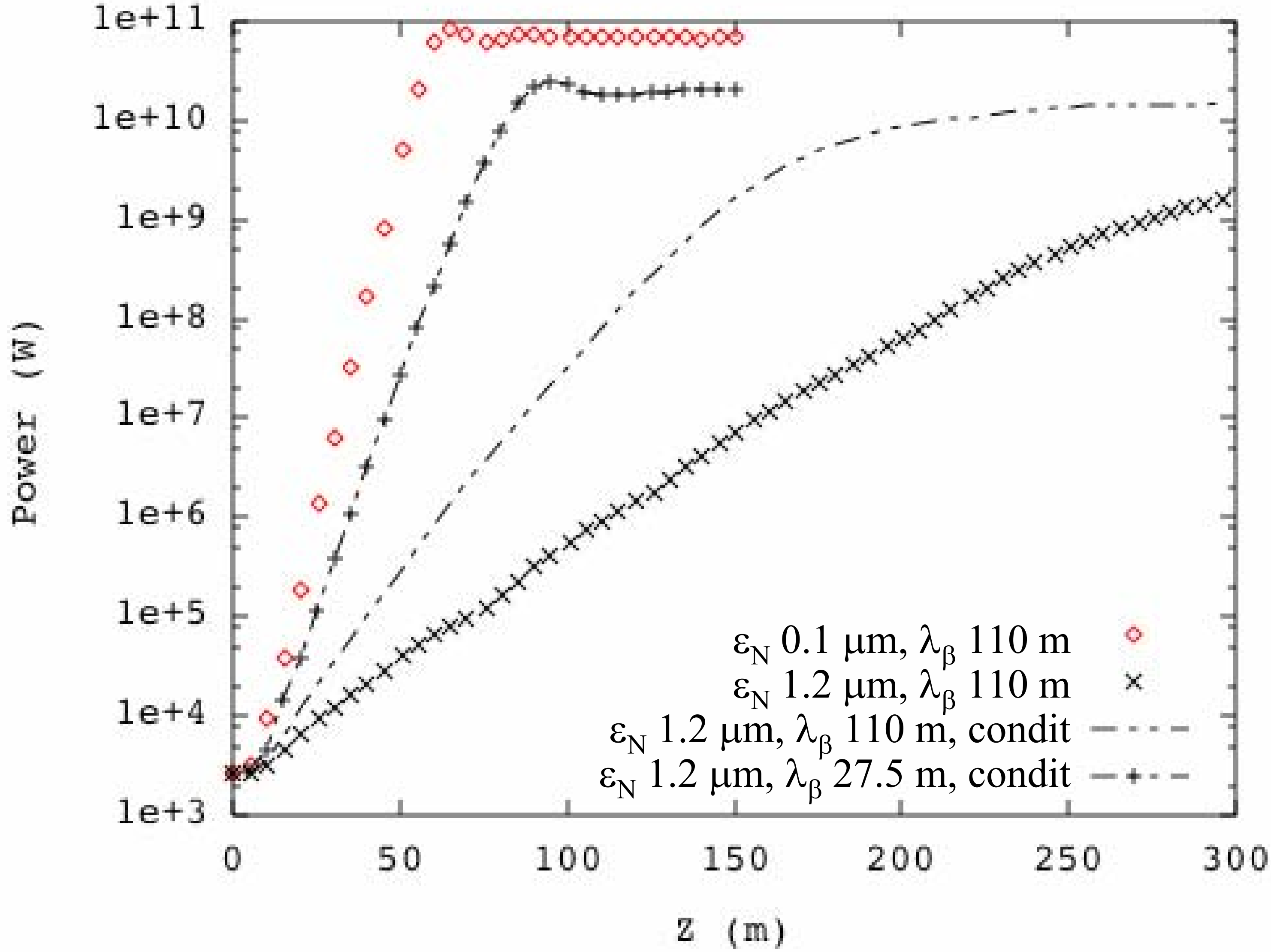
undulator: $\lambda_w = 3$ cm, K = 2.62

Low energy parameters:

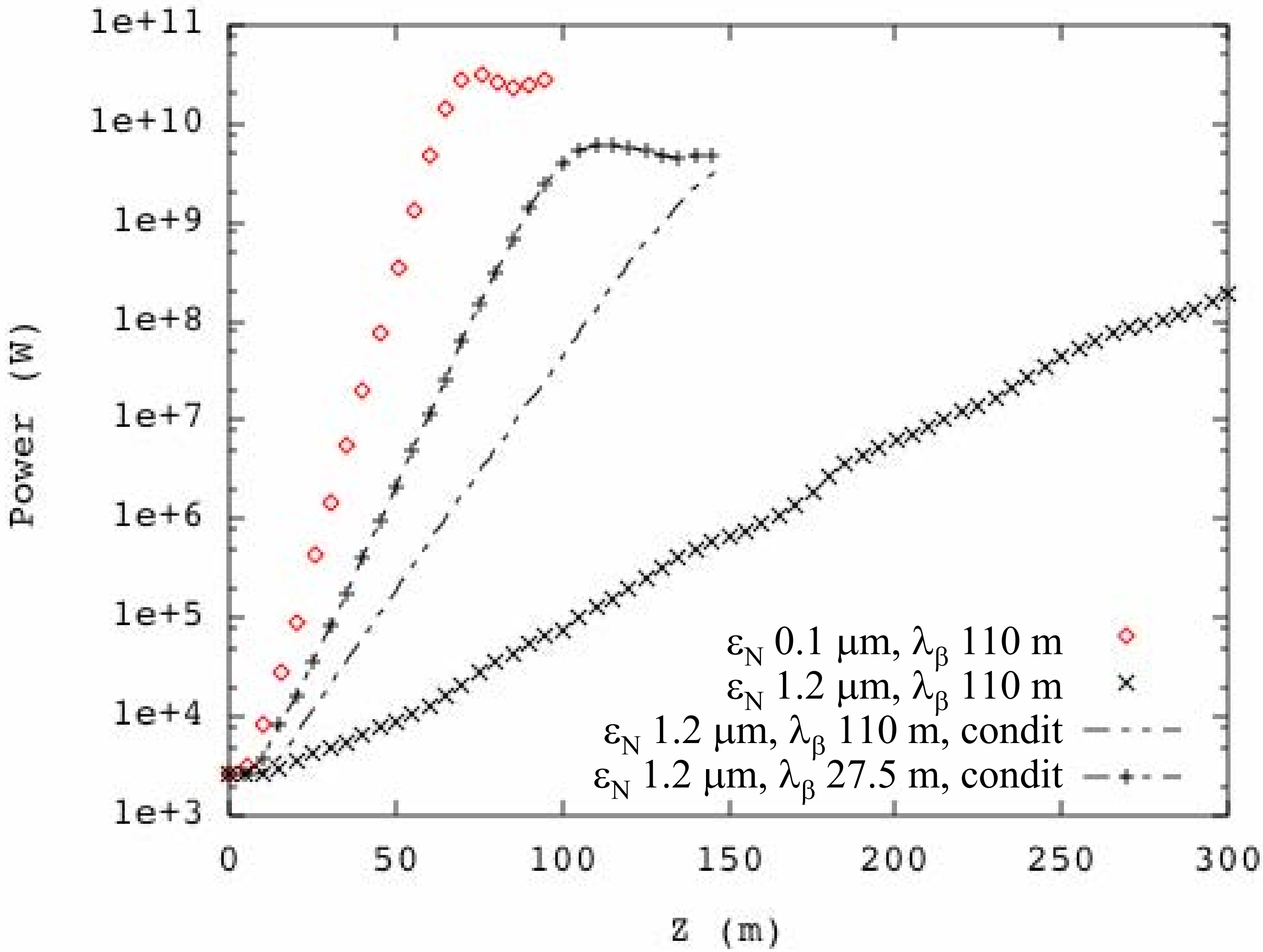
12.1 GeV, $\Delta\gamma/\gamma = 1.2 \times 10^{-4}$

undulator: $\lambda_w = 3$ cm, K = 0.71

Greenfield FEL at 28 GeV



Greenfield FEL at 12 GeV



5. Beam Dynamics in Conditioning Systems

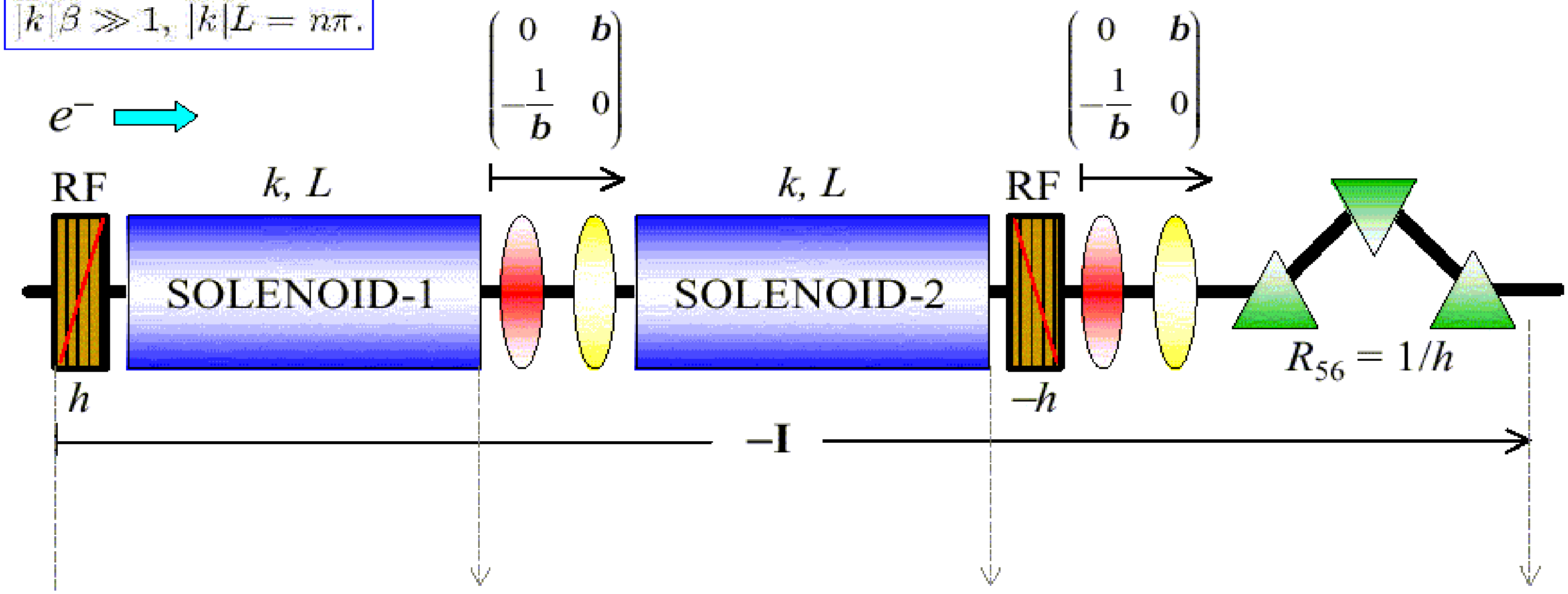
The original suggestion was for very short bunches. That is, we were aware of the Panofsky-Wentzel Theorem. We knew that a long bunch would suffer deflections, but that conditioning would be perfect for a deltya function in z bunch. We thought the Panofsky-Wentzel theorem could be gotten around. (The method we suggested was wrong.)

Others soon showed that in a simple system the deflections were very large and un-acceptable and they speculated that one could never get around the theorem and therefore that conditioning would never work. BUT one can get around the theorem!

Courtesy of P. Emma and G. Stupakov

‘Two-Phase’ FEL Conditioner

$|k|\beta \gg 1, \; |k|L = n\pi.$



$(x_0, x_0', y_0, y_0', z_0, \delta_0)$

$$x_1 \approx x_0 \;, \quad x_1' \approx x_0' + k^2 L h_1 z_0 x_0 \;, \quad z_1 \approx z_0 - \frac{1}{2}k^2 L(x_0^2 + y_0^2) \;, \quad \delta_1 \approx h_1 z_0 \;,$$

$$x_2 \approx \beta x_0' + k^2 L \beta h_1 z_0 x_0 \;, \quad x_2' \approx -\frac{1}{\beta} x_0 + k^2 L \beta h_1 z_0 x_0' \;, \quad z_2 \approx z_0 - \frac{1}{2}k^2 L[x_0^2 + (\beta x_0')^2 + y_0^2 + (\beta y_0')^2] \;, \quad \delta_2 \approx h_1 z_0 \;.$$

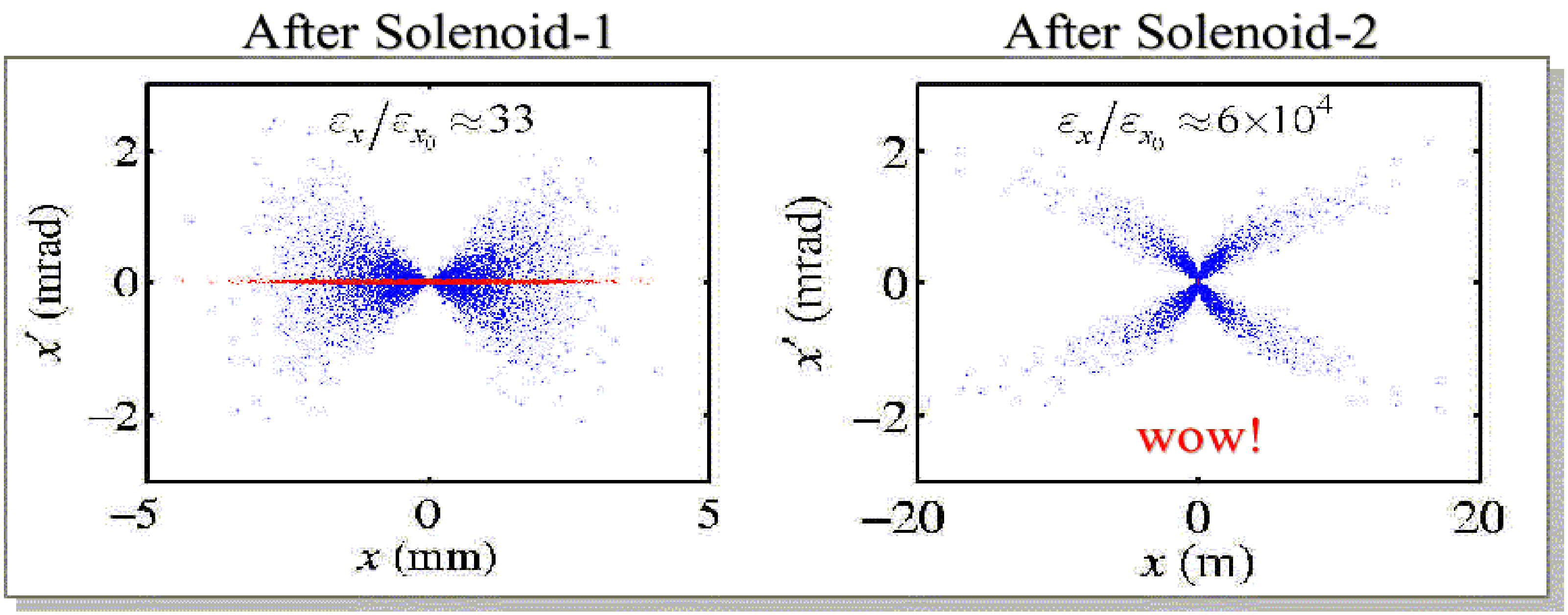
$$x \approx -x_0 - k^2 L \beta^2 h z_0 x_0' \;, \quad x' \approx -x_0' + k^2 L h z_0 x_0 \;, \quad z \approx z_0 + R_{56}\delta_0 \;, \quad \delta \approx \frac{1}{2} h k^2 L[x_0^2 + (\beta x_0')^2 + y_0^2 + (\beta y_0')^2]$$

(see also N. Vinokurov; NIM A **375**, 1996, pp. 264-268)

Courtesy of P. Emma and G. Stupakov

Particle Tracking Through Solenoid System

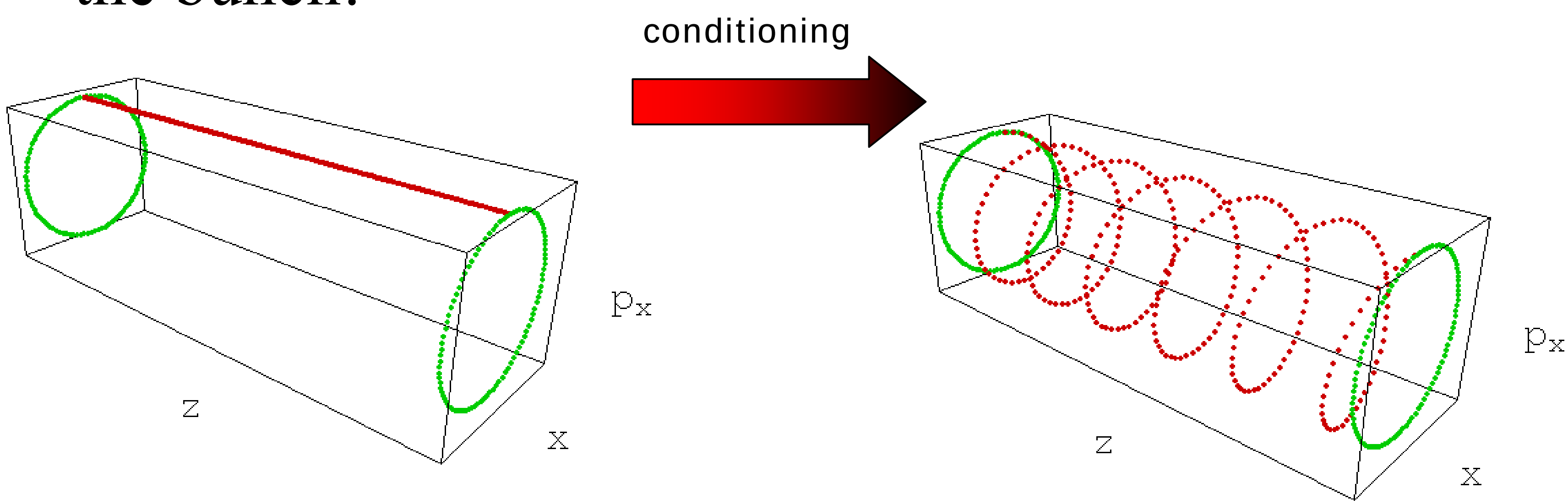
With $k = \frac{k_0}{1+\delta}$



If this is a general result, then conditioning a short wavelength FEL looks **impossible.**

There are extreme effects in transverse phase space

- $\Delta\phi = \mu + \kappa z$
- For LCLS, $\kappa \approx 5\ \mu m^{-1}$
  So for $\sigma_z \approx 20\ \mu m$, there is a *variation* in phase advance of order **100 radians** along the length of the bunch!



The Hamiltonian for an ideal conditioner is straightforward

Next six viewgraphs
From Andy Wolski

$$H = \frac{\mu}{L} J + \frac{\kappa}{L} zJ$$

J and *z* are conserved

$$\Delta\phi = \mu + \kappa z$$

$$\Delta\delta = \kappa J$$

- Since *J* is conserved, the effective emittance of the bunch is preserved
- Since *z* is conserved, the bunch length is preserved
- There is a phase advance that depends upon *z*
- κ is the conditioning parameter:

$$\frac{\Delta\gamma}{\gamma J} = \kappa = \frac{1}{2\beta_u} \frac{\lambda_u}{\lambda_r}$$

How do we construct a conditioning beamline?

- The Hamiltonian for a linear,“smoothly focusing” beamline may be written:

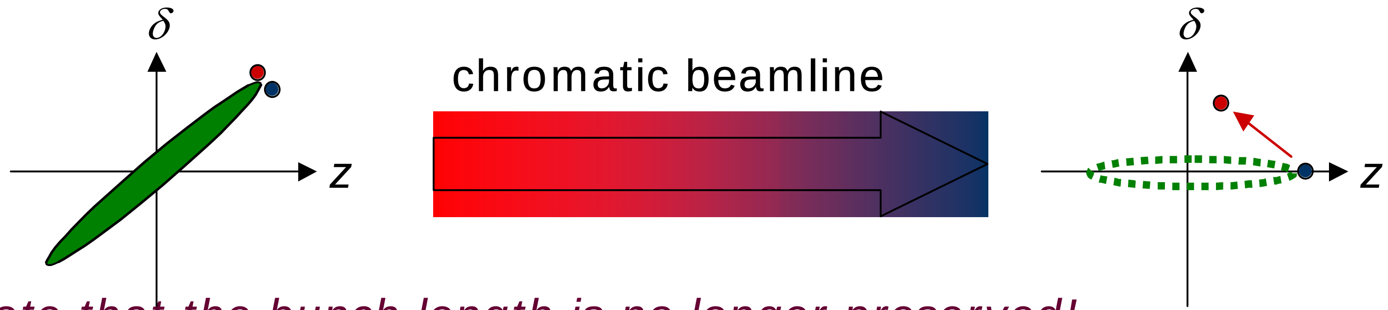
$$H = \frac{\mu}{L} J + 2\pi \frac{\xi}{L} \delta\, J$$

phase advance (per unit length)

chromaticity (per unit length)

- This gives us $\Delta z = 2\pi\xi J$

♣ Two RF cavities may be used at either end of the beamline to convert Δz to $\Delta\delta$ as follows:

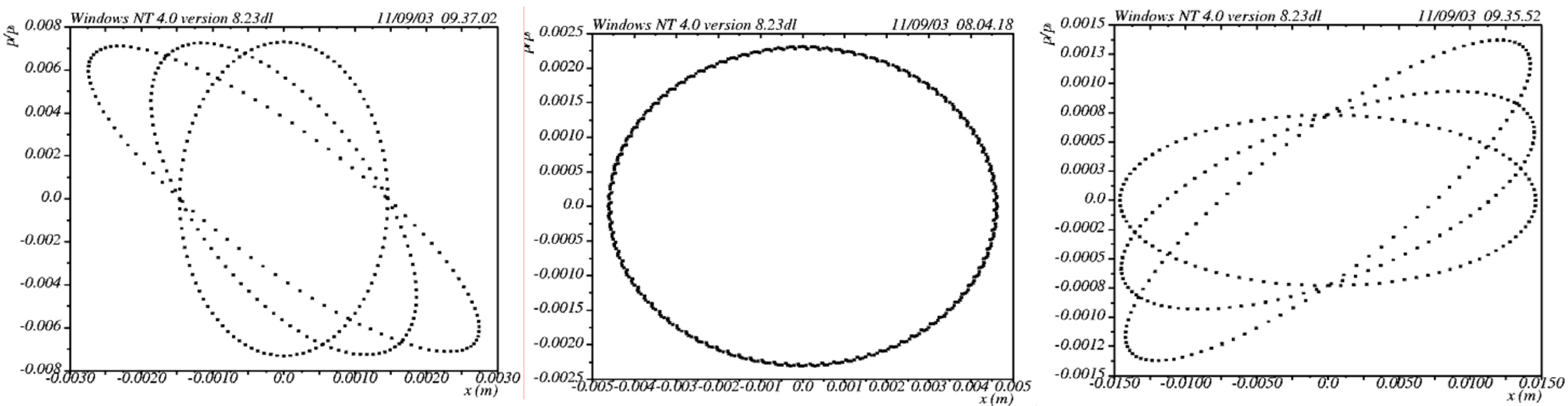


♣ *Note that the bunch length is no longer preserved!*

The beta function can change with energy

$$2J = \gamma x^2 + 2\alpha x p_x + \beta p_x^2$$

- If β varies with energy, then chirping the bunch introduces a phase space mismatch along the bunch.
- The action J of each slice of the bunch is preserved, but…
- …the effective emittance of the whole bunch is blown up.



β = 0.2 m | β = 2 m | β = 20 m

Tracking through solenoid: k_s=1 m^{-1}; k_sL=2π; δ =0, +2.5%, +5.0%

Conditioning can be provided in different ways

- Our beamline analysis shows us how (in principle) to avoid growth of the effective emittance while conditioning; simply have beta at the beginning and end of the channel independent of energy (to first order is enough).
- Different types of conditioner can be used in a properly "matched" beamline, to provide conditioning without growth of effective emittance
  - RF cavities + chromatic "FODO" beamline
  - RF cavities + solenoid
  - TM_{210} mode cavity
  - TM_{110} mode cavity + sextupoles
  - Laser/Wiggler
  - Laser Backscattering
  - Plasma channel

Simple schemes provide small amounts of conditioning

- For a chromatic conditioner:

$$\frac{\Delta\gamma}{\gamma J} = -2\pi\xi \frac{eV_{RF}}{E}\frac{\omega_{RF}}{c}$$

example

$$\xi = -1$$
$$eV_{RF}/E = 0.01$$
$$f_{RF} = 4.8\,\text{GHz}$$
$$\frac{\Delta\gamma}{\gamma J} = 6\times10^{-6}\,\mu\text{m}^{-1}$$

- For a solenoid conditioner:

$$\frac{\Delta\gamma}{\gamma J} = \frac{BL}{(B\rho)}\frac{eV_{RF}}{E}\frac{\omega_{RF}}{c}$$

example

$$BL = 100\,\text{Tm}$$
$$eV_{RF}/E = 0.01$$
$$f_{RF} = 4.8\,\text{GHz}$$
$$\frac{\Delta\gamma}{\gamma J} = 30\times10^{-6}\,\mu\text{m}^{-1}$$

For LCLS, we need 5 μm^{-1}

Specialized cavities also provide "small" conditioning

- For example, TM_{210} mode cavity

$$E_z = \frac{1}{4}\left(\frac{j_{21}}{R}\right)^2 E_0\left(x^2 - y^2\right)\cos(\omega t + \delta)$$

$$P = \frac{m^2 c^5}{e^2}\frac{J_3(j_{21})^2}{16Q}\left(\frac{eE_0}{mc^2}\right)^2 \frac{\omega L}{c} R^2$$

$$\frac{\Delta\gamma}{\gamma J} = \frac{1}{\gamma}\frac{eE_0}{mc^2}\frac{\omega}{c}\beta$$

$P = 1\mathrm{MW}$ $\quad eE_0/mc^2 = 1\,\mathrm{m}^{-1}$

$\gamma = 4000$

$f_{RF} = 5\mathrm{GHz}$

$\beta = 20\mathrm{m}$

$$\frac{\Delta\gamma}{\gamma J} = 6\times 10^{-6}\,\mu\mathrm{m}^{-1}$$

Laser-Wiggler Conditioner

Proposed by Sasha Zholents

Use a laser/wiggler rather than an rf cavity

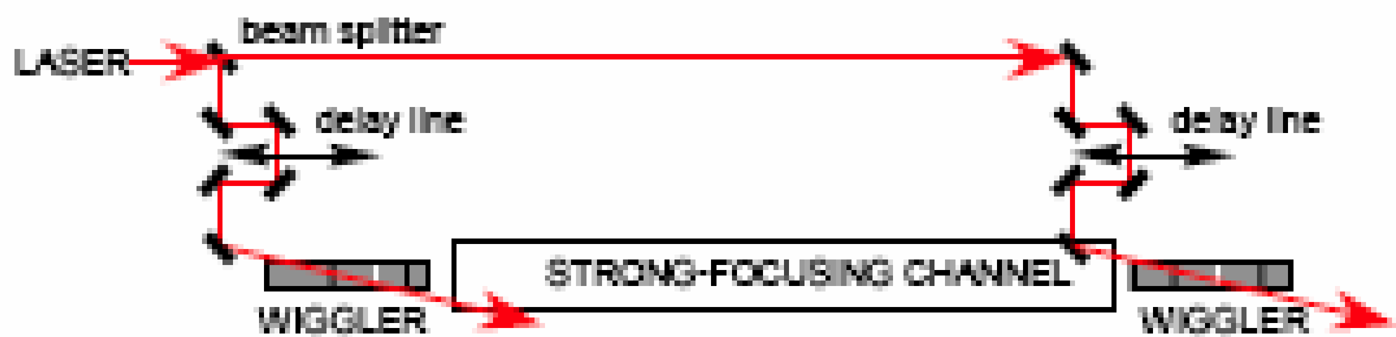
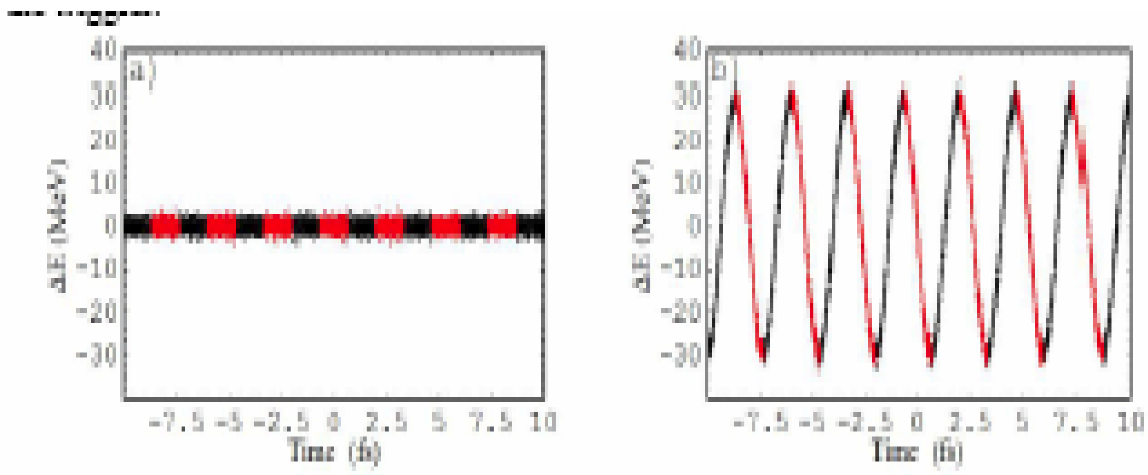


Figure 2. A schematic of the laser assisted conditioner.

Laser-Wiggler Conditioner (Cont)

- Some electrons gain energy and some lose energy. Only about 1/2 are conditioned.



Laser-Wiggler Conditioner (Cont)

- Parameters:
- E=1.5 GeV
- E variation needed = 12 MeV
- Laser: 6 mJ, 800 nm, 100 fs
- Wiggler: 10 periods, 10 cm period
- Non-linear channel: 16 FODO cells, 90 degree advance per period, 25 cm long quads with gradient of 4.3 kG/cm, and total length of 18 m.

Laser-Wiggler Conditioner (Cont)

- Nevertheless, for LCLS at twice its emittance the result is significant:

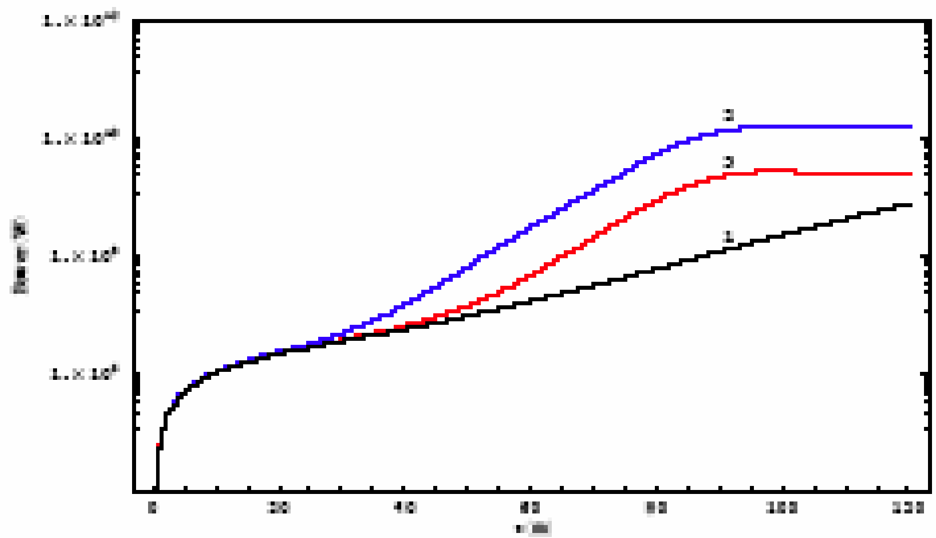
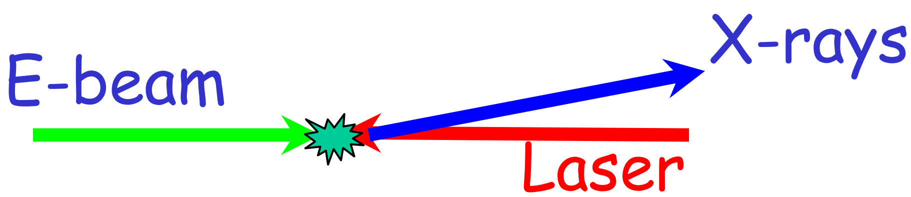


Figure 9. The average x-ray power versus distance for an electron beam with 2.4 mm-mrad normalized emittance: 1 is the case without conditioning, 2 is the case of an ideal conditioning, and 3 is the case of a practical conditioning with a proposed conditioner.

Laser Backscattering Conditioner



- The laser photons are backscattered, Compton scattered, off the moving electrons and thus cause the electron to loose energy. The up shift in energy is $4\gamma^2$. It is easy to tailor the laser pulse so that the electrons near the axis loose more energy than those at larger radii. It is necessary to choose the conditioning energy, $\gamma m_e c^2$, to be sufficiently low that there are a good number, N, of photons, of energy $h\nu$ required to give the necessary conditioning energy shift $\Delta\gamma_c m_e c^2$. We have:

$$N = \frac{\Delta\gamma_c m_e c^2}{h\nu 4\gamma^2}$$

(Independent work by C. Schroeder et al, Phys Rev. Lett. 92, 252301(2004))

Plasma Channel Conditioner

Work by Jonathan Wurtele, Gregg Penn, and myself.
Send a laser through a gas in a tube. Blow out all the electrons and make an ion channel. Send the high energy beam just behind the laser before the slow electrons return.

In the plasma channel $\beta = (2\gamma)^{1/2} c/\omega_p$
where $\omega_p = 6 \times 10^{12} (n\ (cm^{-3})/10^{16})^{1/2}$ and $\lambda_p = 2\pi c/\omega_p$

For example at $n = 10^{17}\ cm^{-3}$ and 1 GeV, $\omega_p = 2 \times 10^{13}\ s^{-1}$,
$1/\ \omega_p = 50$ fs, $\lambda_p = 100 \mu m$, and $\beta = 0.1$ cm

Simulations, to follow, by Gregg Penn. Two cases (similar to Zholents and Emma and Stupakov [a FOFO channel at 100 MeV[)
Clearly at 100 MeV only condition a small time slice.

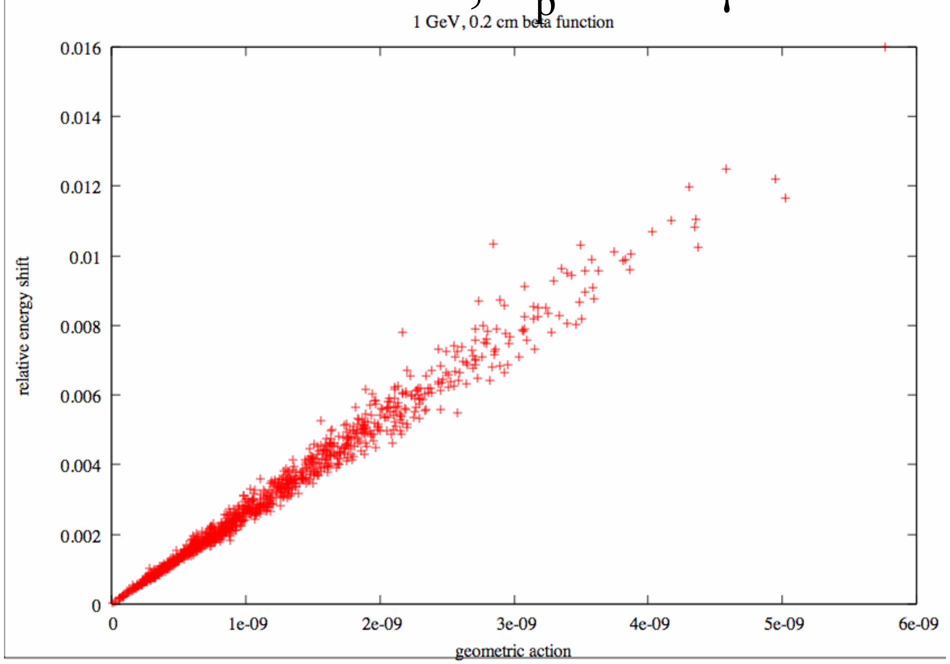
Plasma channel beam conditioner

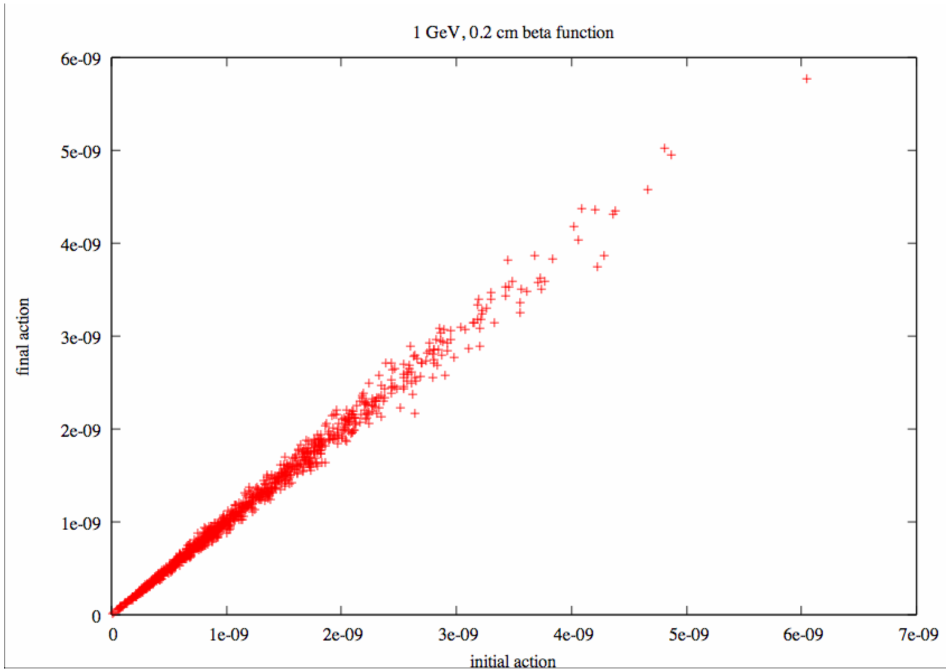
beam energy 1 GeV
beam length 20 micron
beam emittance 1.2 micron
Beam radius 1.1 micron

Plasma channel:
length 5 m, $n_p = 2.8 \times 10^{16}$ cm^{-3}
beta = 0.2 cm, $\lambda_p = 199 \mu m$

RF:
period 300 micron
voltage: 100 MeV
gradient: 42 MeV over 20 micron
(Need laser wake)

Result:
$\delta\gamma/\gamma J = 2.5$ micron^{-1}





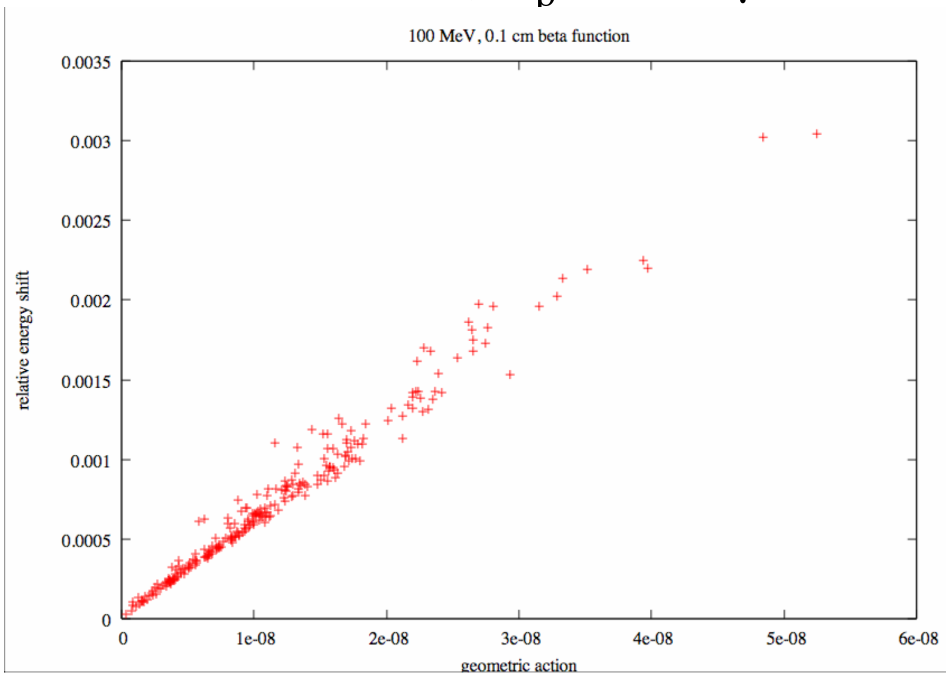
Plasma channel beam conditioner

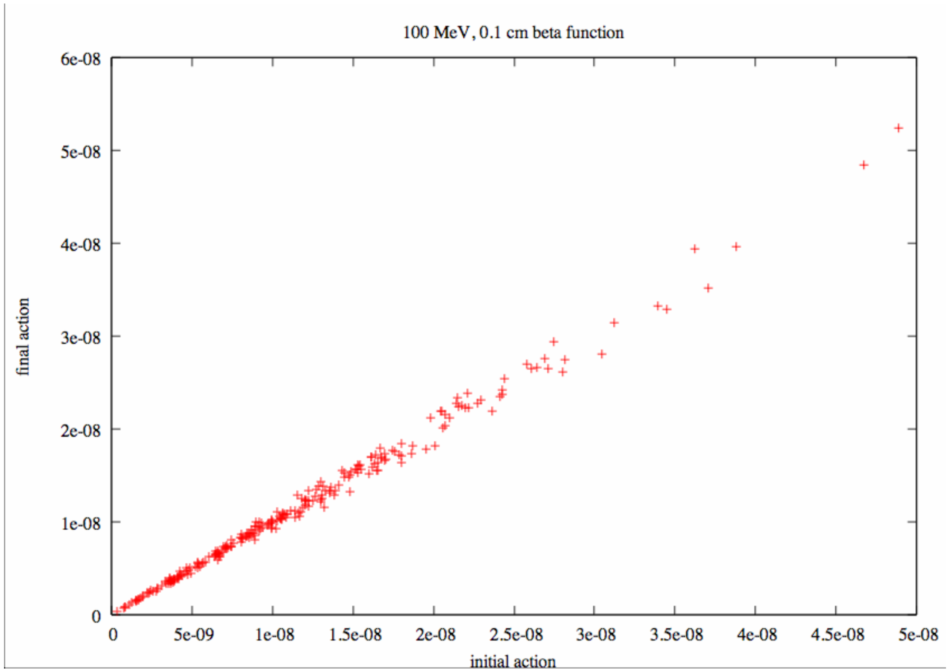
beam energy 100 MeV
beam length 1 mm
beam emittance 1.2 micron
Beam radius 2.4 microns

Plasma channel:
length 2 m, $n_p = 1.12 \times 10^{16} cm^{-3}$
beta = 0.1 cm, $\lambda_p = 315\ \mu m$

RF:
period 10 cm
voltage: 100 MeV
gradient: 7 MeV over 1 mm
(S-band okay)

Result:
$\delta\gamma/\gamma J = 0.06\ micron^{-1}$





6. Emittance Transfer

Work by Jie Wei, Hiromi Okamoto, and myself.

Consider a "Green Field FEL" at 12 GeV, transverse emittance is 1.2×10^{-6} m, and energy spread $\Delta\gamma/\gamma = 1 \times 10^{-4}$, and bunch length $L = 100$ μm. The longitudinal emittance is $\gamma(\Delta\gamma/\gamma)\, L = 2.4 \times 10^{-4}$ m

Notice the very large difference between longitudinal and transverse emittance (Noted by Kwang-Je Kim). Even if we transfer some emittance from the transverse to longitudinal, it will hardly increase longitudinal and very much improve the FEL.

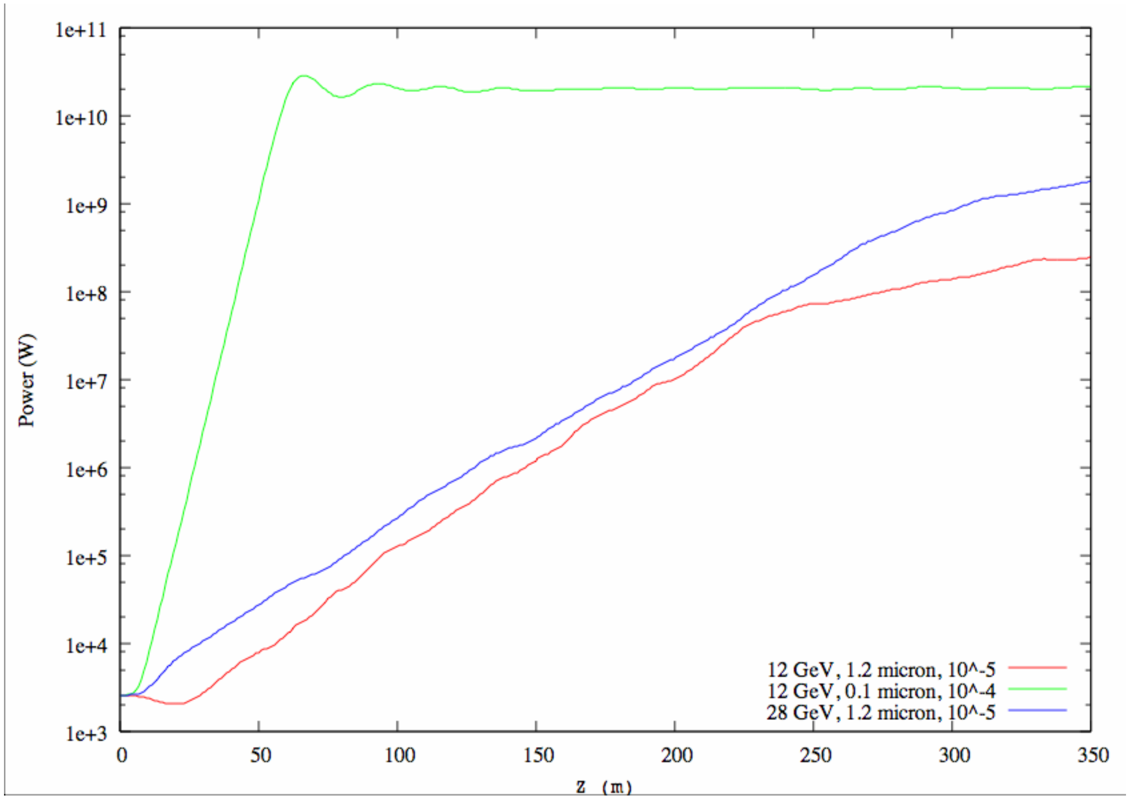
In the next viewgraph Gregg Penn has considered a reduction by a factor of ten in transverse emittance and a factor of ten up in $\Delta\gamma/\gamma$

6. Emittance Transfer (Cont)

Comparing increase in energy vs. decrease in emittance at expense of energy spread.

Curve generated by

Gregg Penn



I=3.5 kA, delta gamma / gamma starts at 10^-5, in case 2 goes up to 10^-4. transverse emittance starts 1.2 micron, in case 2 goes down to 0.1 micron.

20 m avg beta function, 3 cm und. period, 0.4 angstrom rad.

6. Emittance Transfer (Cont)

We desire to reduce the small transverse emittance at the expense of the larger longitudinal emittance.

Can that be done? It is easy to make the larger smaller and the smaller larger, but we want the reverse!

So we took a usual emittance exchange case and studied it in detail and, in this way learned what is needed.

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6. Emittance Transfer (Cont)

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

6. Emittance Transfer (Cont)

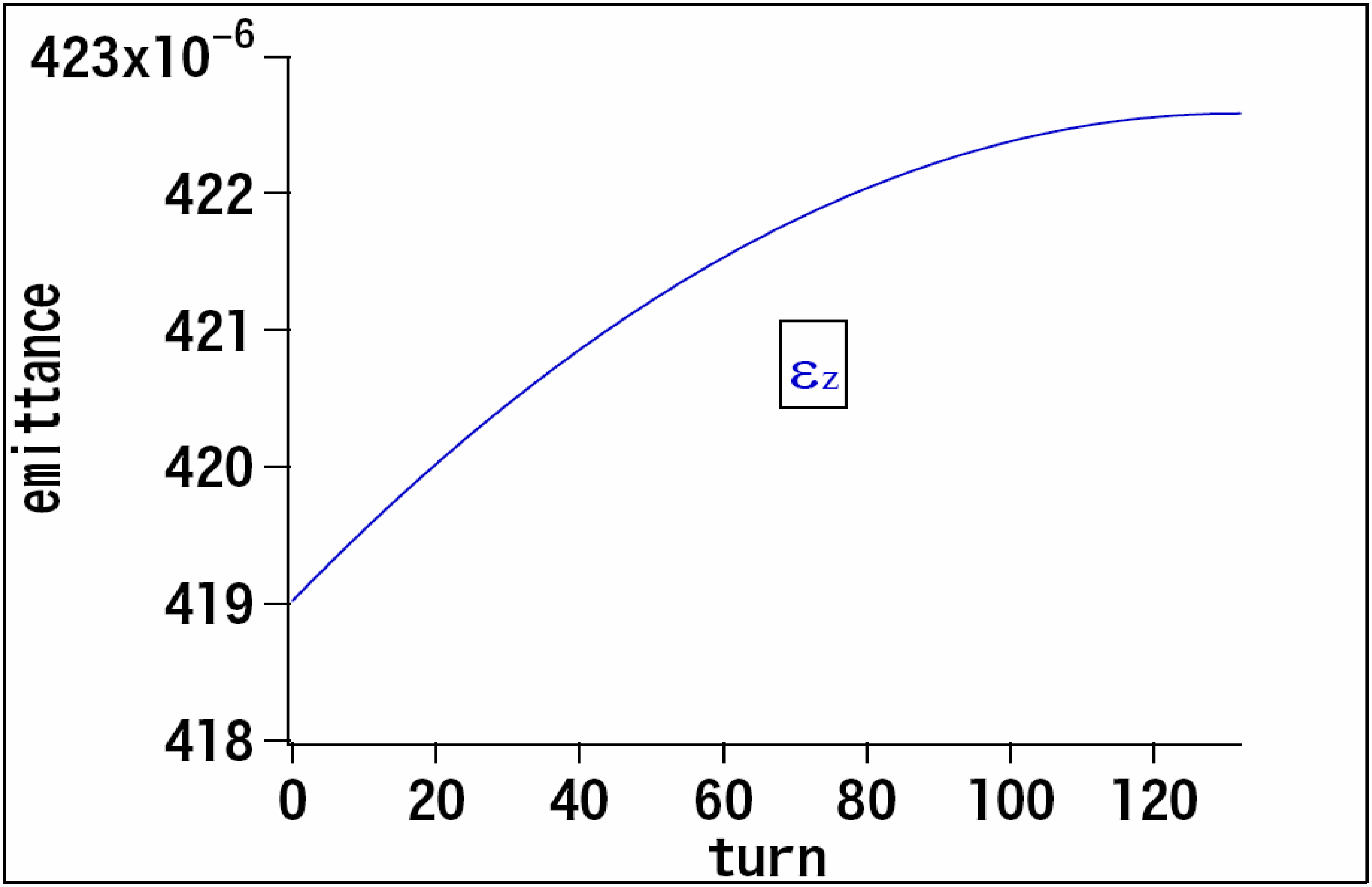
QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

6. Emittance Transfer (Cont)

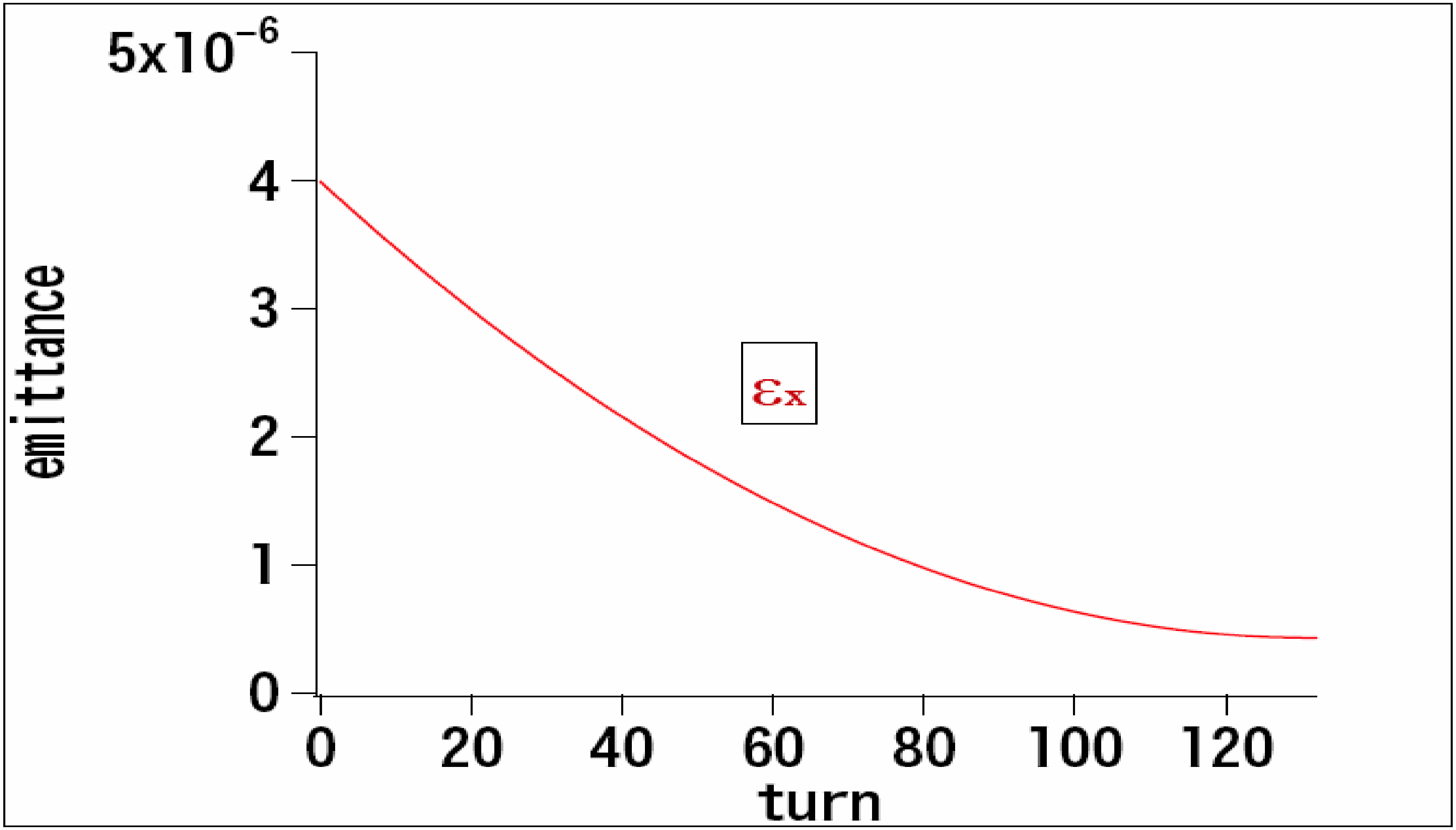
Using the insight we have developed, we now started emittance transfer from small to large (smaller getting smaller), by introducing the necessary correlations at $t = 0$. Namely, we want p_x correlated with z and p_z correlated with x.

In the next viewgraphs is a simulation done by Hiromi Okamoto where the emittance was 1 (x) and 100 (z) and $\nu_x = 5.1$ and $\nu_z = 0.1$ and the coupling potential was $0.01xz$.

6. Emittance Transfer (Cont)

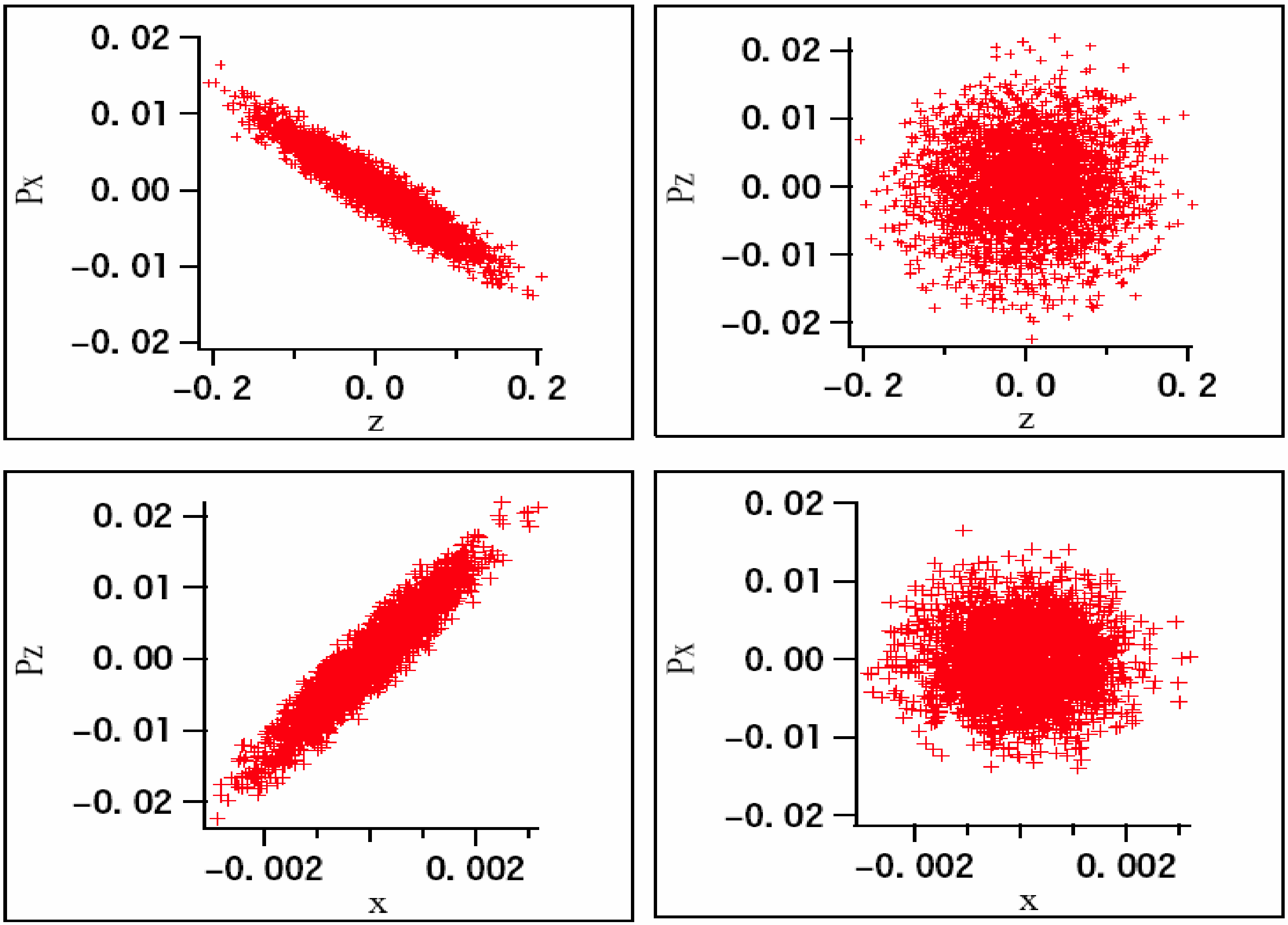


6. Emittance Transfer (Cont)



6.Emittance Transfer (Cont)

The initial distribution that is needed:



7. Conclusions

for

Transverse-Longitudinal Correlations: FEL

Performance and Emittance Exchange

- If conditioning can be achieved it would have a very large impact on FEL performance.

  (Li-Hua Yu, Whittum).
- Conditioning without growth of effective emittance is possible in a symplectic system (Vinokorov, Wolski).
- It appears to be difficult (but maybe not impossible) to achieve the amount of conditioning likely to be required by real FELs (Kim, Emma, Wolski et al).
- Non-conventional (laser/wiggler (Zholents), laser backscattering (Schroeder), and laser-plasma (Wurtele and Penn) conditioning holds promise.

7. Conclusions
for
Transverse-Longitudinal Correlations: FEL Performance and Emittance Exchange

- Emittance transfer would benefit x-ray FELs (Kim)
- Emittance transfer from a large emittance to a small emittance is possible (Wei and Okamoto)
- Practical emittance transfer schemes have yet to be developed (but no one has even tried yet).

The End

Thank you for your attention