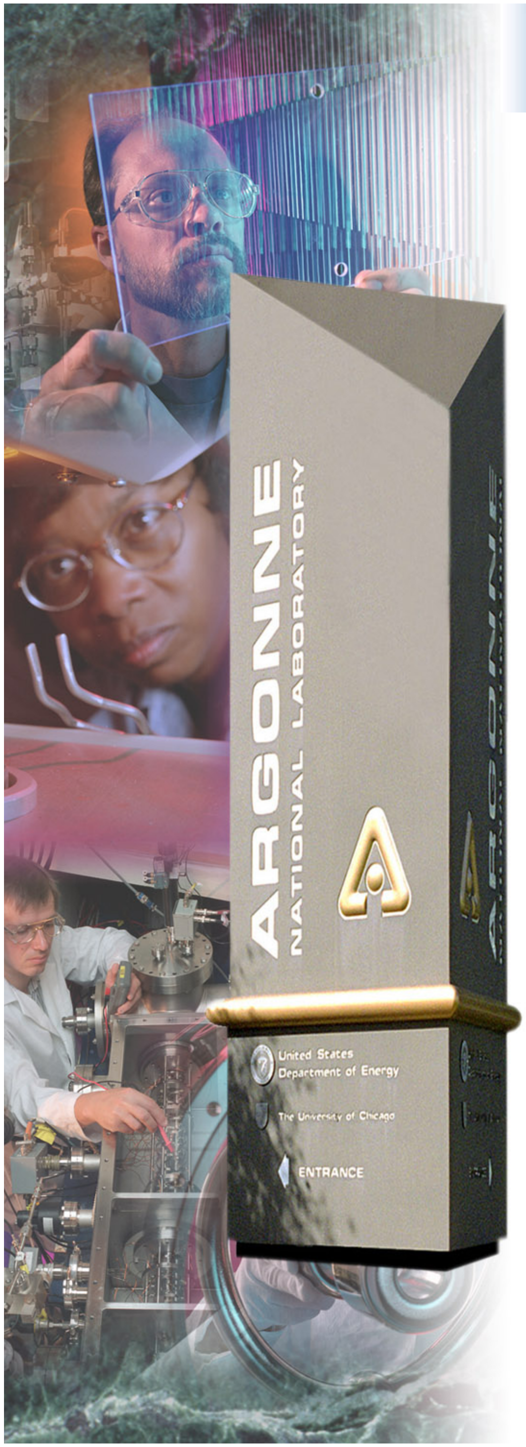

ARGONNE NATIONAL LABORATORY

Transverse-Longitudinal Phase Space Manipulations

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Argonne National Laboratory
and The University of Chicago

COOL05

International Workshop on Beam Cooling and Related Topics

September 18-23, 2005

Eagle Ridge, Galena, Illinois

THE UNIVERSITY OF CHICAGO
Office of Science
U.S. DEPARTMENT OF ENERGY

Phase Space Manipulation

- **Accelerators are for manipulation of beams in 6D phase space**
- **Phase space manipulation in accelerators has been mainly in one 2D subspaces (z, δ) or (x, x') and in Hamiltonian system**
- **Beam cooling is an advanced phase space manipulation in weakly *non-Hamiltonian* system**
- **Manipulation involving 6-D phase space of *Hamiltonian* system can greatly enhance accelerator performance**
- **This and the next talk are about *Hamiltonian* phase space manipulation involving more than 2D**

Contents

- Some properties of Hamiltonian Transport
- Transverse-longitudinal *switching* for x-ray FELs
- Flat beam technique
- Applications of flat beam
  - Smith-Purcell FEL
  - X-ray pulse compression

Beam Transport and Manipulation

- **6D phase space: (x, x´, y, y´, z, δ)**
- **We will use 4D for notational simplicity**
- $$\mathbf{X} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$$
- **Beam matrix:**

$$\sum = \langle x\tilde{x} \rangle = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle xx' \rangle & . & . & . \\ \langle xy \rangle & . & . & . \\ \langle xy' \rangle & . & . & \langle y_1{}^2 \rangle \end{bmatrix}$$

- **Transfer matrix: M**

  $X = M X_o$, $\Sigma = M \Sigma_o \tilde{M}$

Hamiltonian Transport

- **Unit symplectic matrix**

$$J = \begin{bmatrix} J_{2D} & 0 \\ 0 & J_{2D} \end{bmatrix}, \quad J_{2D} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- **M is symplectic:** $\tilde{M}\,J\,M = J$

- ***Det M = 1***

- **There are six constraints on 2D submatrices of M**

Conserved Emittances

- **2D-case (x, x´):**

$$\sum_{2D} = \varepsilon_{2D}\, D \begin{bmatrix} \beta & 0 \\ 0 & 1/\beta \end{bmatrix} \tilde{D} \qquad \varepsilon_{2D}{}^2 = det \sum_{2D} = \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2$$

- **4D-case (x, x´, y, y´):**
  - $\varepsilon_{4D}{}^2 = det \sum$ is conserved
  - **For *uncoupled* case, can find a symplectic transformation**
  - $$\sum \rightarrow \sum^s = \begin{bmatrix} \varepsilon_a T_a & 0 \\ 0 & \varepsilon_b T_b \end{bmatrix}$$

$I_2 = \varepsilon_a{}^2 + \varepsilon_b{}^2$ is conserved

Emittance Switching Theorem

(E. Courant, …)

- **ε_a and ε_b are uniquely determined up to switching.**

$$(\varepsilon_a, \varepsilon_b) \longrightarrow (\varepsilon_a, \varepsilon_b) \text{ or } (\varepsilon_b, \varepsilon_a)$$

- Proof:

$$\varepsilon_{1a}^{2} + \varepsilon_{1b}^{2} = \varepsilon_{2a}^{2} + \varepsilon_{2b}^{2}$$

$$\varepsilon_{1a}^{2}\varepsilon_{1b}^{2} = \varepsilon_{2a}^{2}\varepsilon_{2b}^{2}$$

$$\therefore \quad QED$$

Projected Emittances for Coupled Cases

$$\Sigma = \begin{pmatrix} \Sigma_x & C \\ \tilde{C} & \Sigma_y \end{pmatrix}, \quad C \neq 0 \text{ (coupled)}$$

$$\varepsilon_x{}^2 = \det \Sigma_x,\ \varepsilon_y{}^2 = \det \Sigma_y : \quad \text{projected emittances}$$

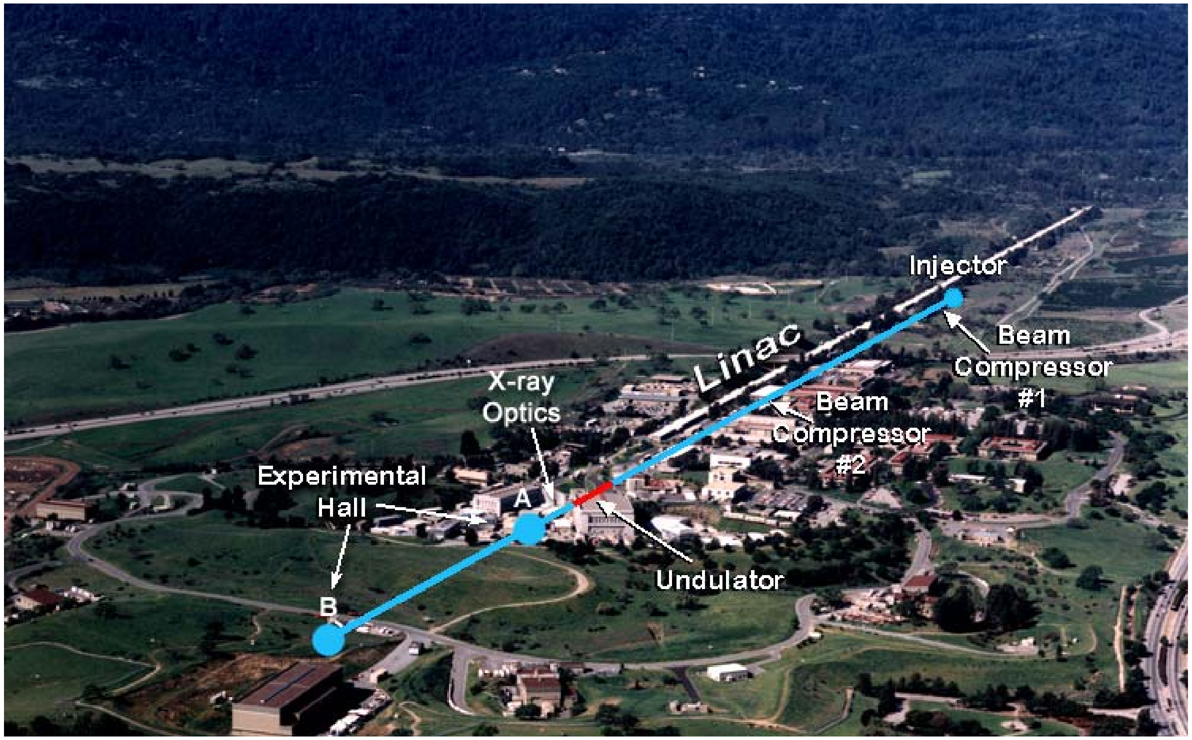
- **Projected emittances are not conserved**
- **Some properties:** (K.L. Brown & R.V. Servanckx, SLAC-PUB 4679 (1989))
  - Start from uncoupled emittances ε_{xo} and ε_{yo}
  - If $\varepsilon_{xo} = \varepsilon_{yo} \Rightarrow \varepsilon_x = \varepsilon_y$ for all s

    $$\varepsilon_x \geq \varepsilon_{xo}$$
  - If $\varepsilon_{xo} \neq \varepsilon_{yo}$

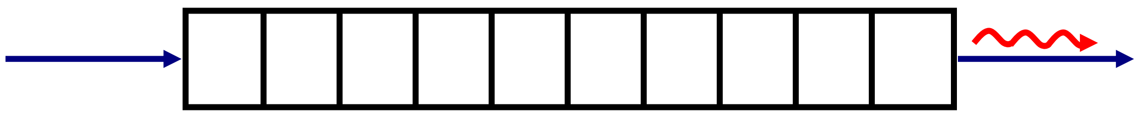
    $$\varepsilon_x + \varepsilon_y \geq \varepsilon_{xo} + \varepsilon_{yo}$$
- **Useful applications appear difficult, but …**

  (A. Sessler's talk)

Emittance Switching for X-Ray FELs



SASE FEL for 30 keV



- LCLS reference parameters:

  λ = 8 keV, λ_u = 3 cm, K = 3.7, I_p = 3.5 kA, E_e = 15 GeV,

  $\Delta E/E = 10^{-4}$ (2x10^{-6} possible) ε_n = 1.2 mm-mrad, L_{sat} = 100 m

- Vary K, ε_n, and E_e (Z.R. Huang)

K	E_e (GeV)	ε_n (mm-mrad)	L sat (m)
3.7	30	1.2	300
3.7	30	0.5	130
3.7	30	0.1	40
1	12	0.1	60

← **shorter undulator** (row: 3.7, 30, 0.1, 40)

← **shorter undulator and shorter linac** (row: 1, 12, 0.1, 60)

- ***It pays to strive for an ultralow emittance e-beam***

An Emittance Switching Scheme for Improved X-Ray FEL Performance

(P. Emma, Z. Huang, P. Piot, and KJK)

- **Flat beam technique (units in m-rad)**

$$\gamma\varepsilon_x \otimes \gamma\varepsilon_y : \quad \left(10^{-6}\right)^2 \rightarrow 10^{-5} \otimes 10^{-7}$$

- **Use short electron beam σ_z = 20 μ**

$$\gamma\varepsilon_z = \sigma_z \sigma_\gamma = 20\mu \otimes 5\times 10^{-3} = 10^{-5}$$

Q = 15 pC, I = 90 A

- **Switch (x $\leftrightarrow$ z)**

$$\gamma\varepsilon_x \otimes \gamma\varepsilon_y \otimes \gamma\varepsilon_z \rightarrow \left(10^{-7}, 10^{-7}, 10^{-5}\right)$$

$$Partition: \sigma_z = 3.7\times 10^{-6}, \delta\gamma = 2.7 \quad \left(\gamma\varepsilon_z = 10^{-5}\ \mathrm{m-rad}\right)$$

$$\Rightarrow I = 90\times 20/2.7 = 500\mathrm{A}, \delta\gamma/\gamma = 10^{-4} @ 15\ \mathrm{GeV}$$

Emittance Exchange: Transverse to Longitudinal

transverse RF in a chicane (P.Emma)

$$B_y = \frac{E_0}{ac} z, \quad B_x = B_z = 0$$

Electric and magnetic fields

$$E_z = E_0 \frac{x}{a}, \quad E_x = E_y = 0$$

$$\eta k = 1$$

k

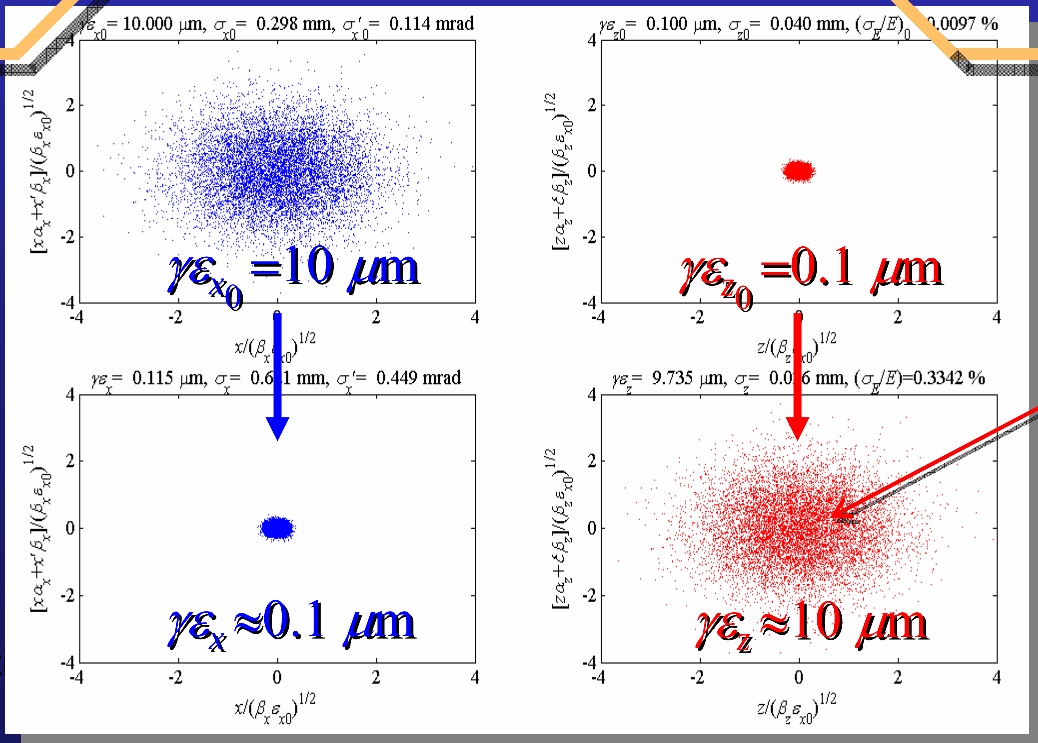
η

ε_{x_0}

ε_{z_0}

$\varepsilon_x \approx \varepsilon_{z_0}$

$\varepsilon_z \approx \varepsilon_{x_0}$



Cross-term and 2nd-order dispersion limit exchange

system also compresses bunch length

SLAC-PUB-9225, May 2002

Flat Beam Technique

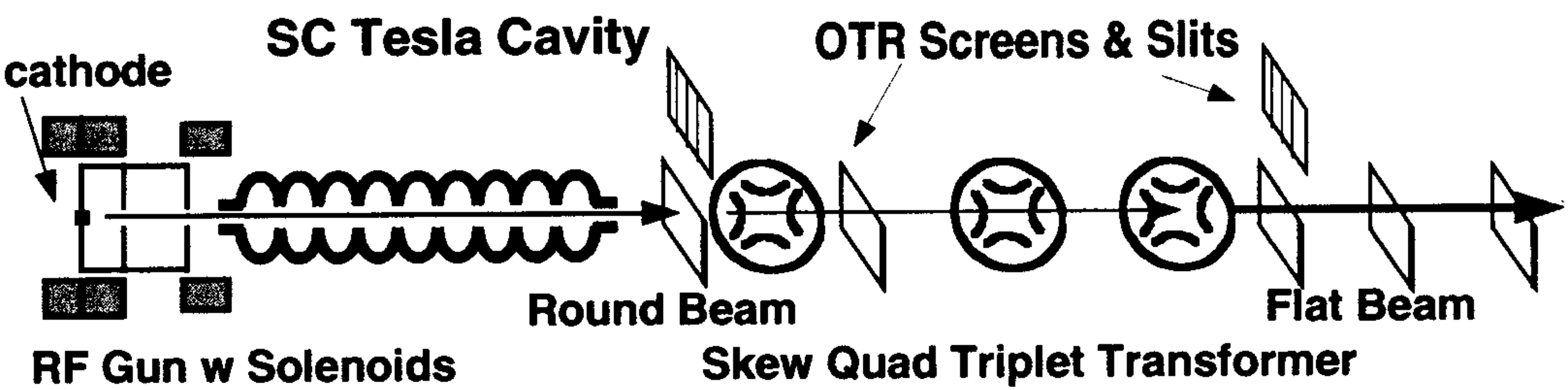
Theory

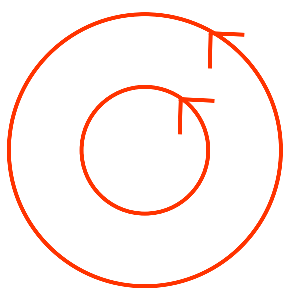
- Y. Derbenev (1998)
- R. Brinkmann, Ya Derbenev, and K. Floettmann(2001)
- A. Burov, S. Nagaitsev, Ya Derbenev (circular basis)
- KJK

Experiment at FNAL A0

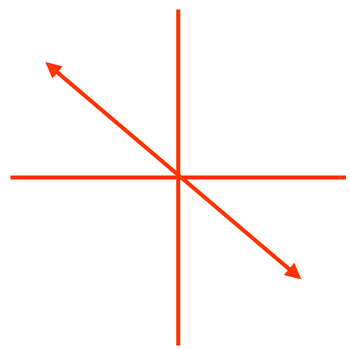
- D. Edwards, H. Edwards, Ph. Piot,…
- Yin-e Sun (U of C thesis, May 2005)

Schematics of Flat Beam Experiment at FNPL





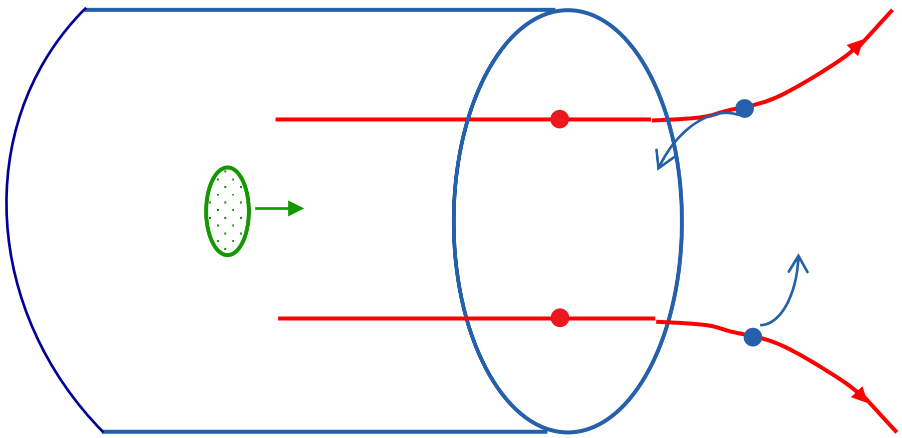
$$\begin{cases} x = A \cos \kappa z \\ y = A \sin \kappa z \end{cases} \longrightarrow \begin{cases} A \cos \kappa z \\ A \sin (\kappa z + \pi/2) \end{cases}$$



vortex beam

flat beam

Fringe of Solenoidal Field Producing Kinetic Angular Momentum



Cathode Immersed in Solenoidal Field

- Motion in solenoidal field is most conveniently described in a rotating (Larmor) frame

$$\frac{d\theta}{ds} = \frac{qB(s)}{2P_s}$$

- Particle coordinates right after cathode plane

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{s=0^+} = \begin{pmatrix} x \\ y' - \kappa_o x \\ y \\ x' + \kappa_o y \end{pmatrix}$$

(x´,y´): thermal angular spread

$$\kappa_o = \frac{qB}{2mc}$$

$$\Sigma = \begin{bmatrix} \varepsilon_{eff} T_o, & \mathcal{L} J \\ -\mathcal{L} J, & \varepsilon_{eff} T_o \end{bmatrix}, \quad T_o = \begin{bmatrix} \beta & 0 \\ 0 & 1/\beta \end{bmatrix}$$

$$\beta = \frac{\sigma_c}{\sqrt{\sigma_c'^2 + \kappa_o^2 \sigma_c^2}}$$

$$\mathcal{L} = \langle xy' - yx' \rangle / 2 = \kappa_o \sigma_c^2$$

$$det \sum \Big|_{s=0+} = \varepsilon_{4D}{}^2 = \left(\varepsilon_{eff}{}^2 - \mathcal{L}^2\right)^2$$

$$\varepsilon_{4D}{}^2 = \varepsilon_{th}{}^4$$

(even if transform before → after cathode surface is not symplectic)

$$\therefore \varepsilon_{eff} = \sqrt{\mathcal{L}^2 + \varepsilon_{th}{}^2}$$

- After triplet transformation:

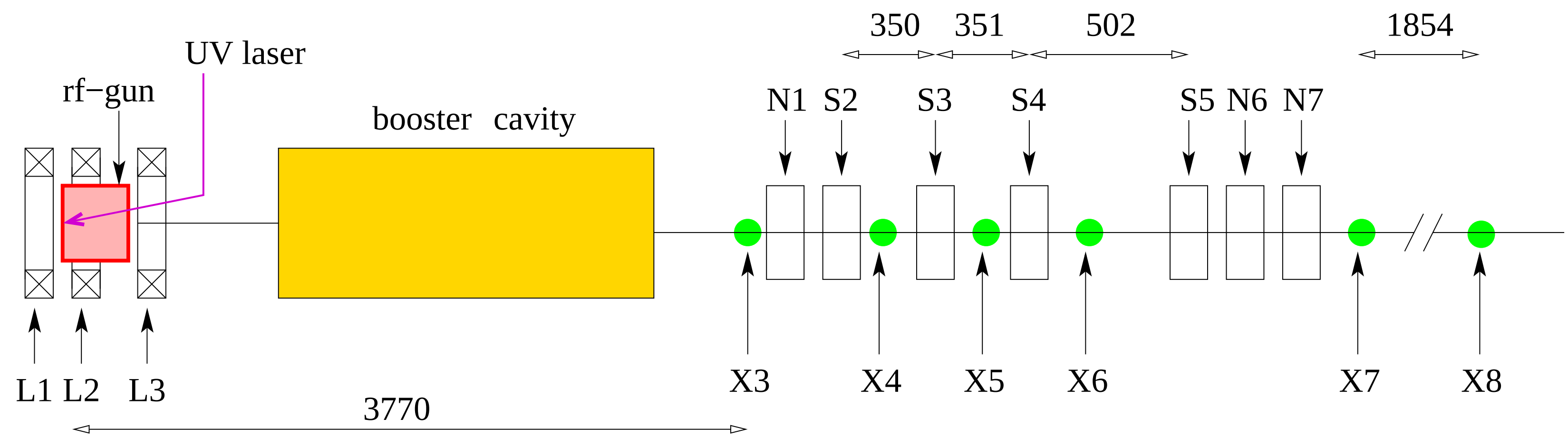
$$\sum \rightarrow \begin{bmatrix} \varepsilon_+ T_+ & 0 \\ 0 & \varepsilon_- T_- \end{bmatrix} \quad T_\pm = \begin{bmatrix} \beta_\pm & 0 \\ 0 & 1/\beta_\pm \end{bmatrix}$$

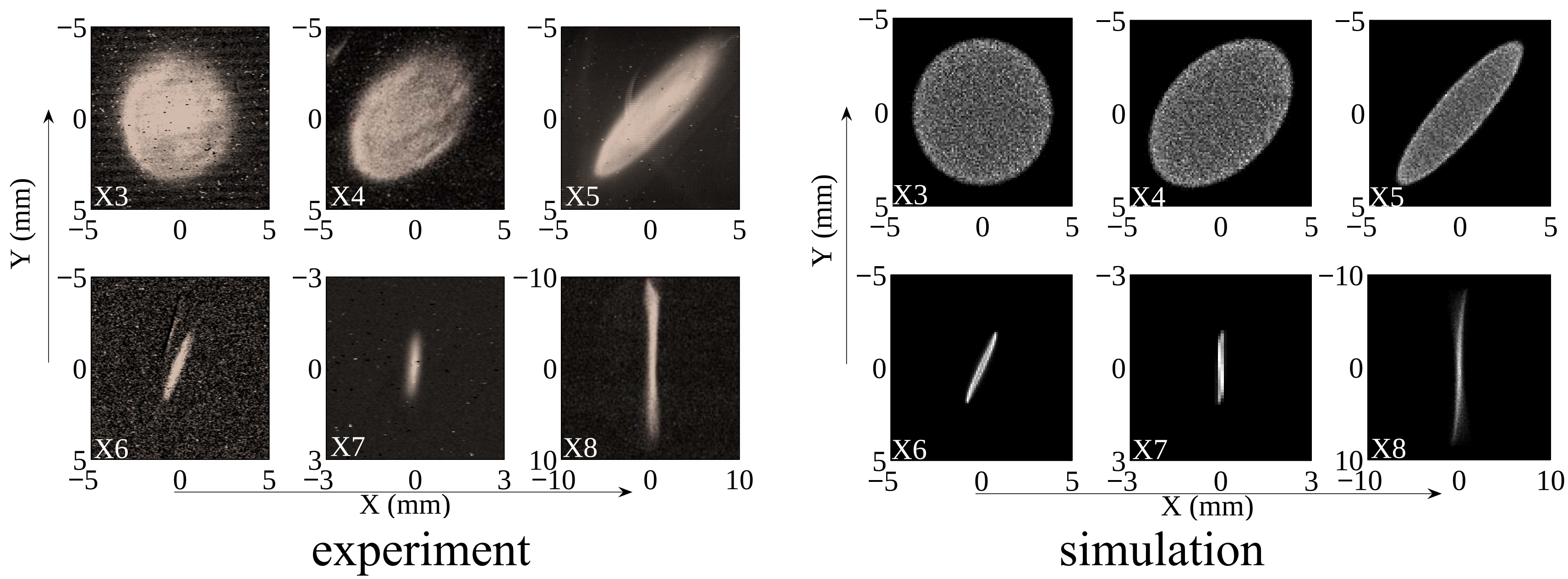
- $\varepsilon_\pm = \varepsilon_{eff} \pm \mathcal{L}$

$$\therefore \varepsilon_+ \varepsilon_- = \varepsilon_{th}{}^2$$

$$\frac{\varepsilon_+}{\varepsilon_-} = \frac{\varepsilon_{eff} + \mathcal{L}}{\varepsilon_{eff} - \mathcal{L}} \approx \left(\frac{2\mathcal{L}}{\varepsilon_{th}}\right)^2, \quad \text{for } \mathcal{L} >> \varepsilon_{th}$$

Removal of angular momentum and generating a flat beam





Applications of Flat Beams

- **Smith-Purcell FEL**

  Also, image charge undulator (Ya Derbenev)

- **Compression of x-rays to pico-femtosecond pulses**

Smith-Purcell Radiation

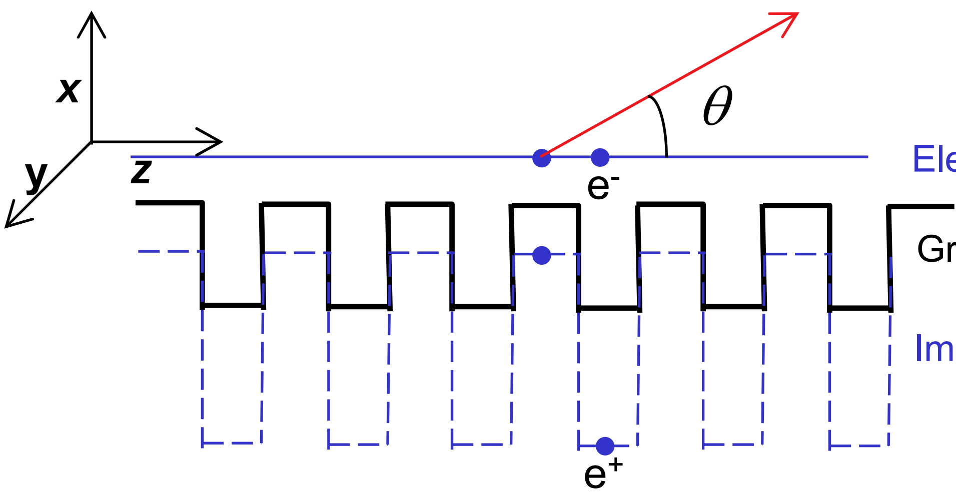
- An electron beam traveling close and parallel to a metallic reflection grating

Dartmouth parameters**

λ_g = 173 μm, β = 0.35 (35 keV)

d = 100 μm, w = 62 μm,

b = 10 μm, L = 12.7 mm



(Assume translational symmetry in y-direction)

$$\lambda = \frac{\lambda_g}{\beta}(1-\beta\cos\theta)$$

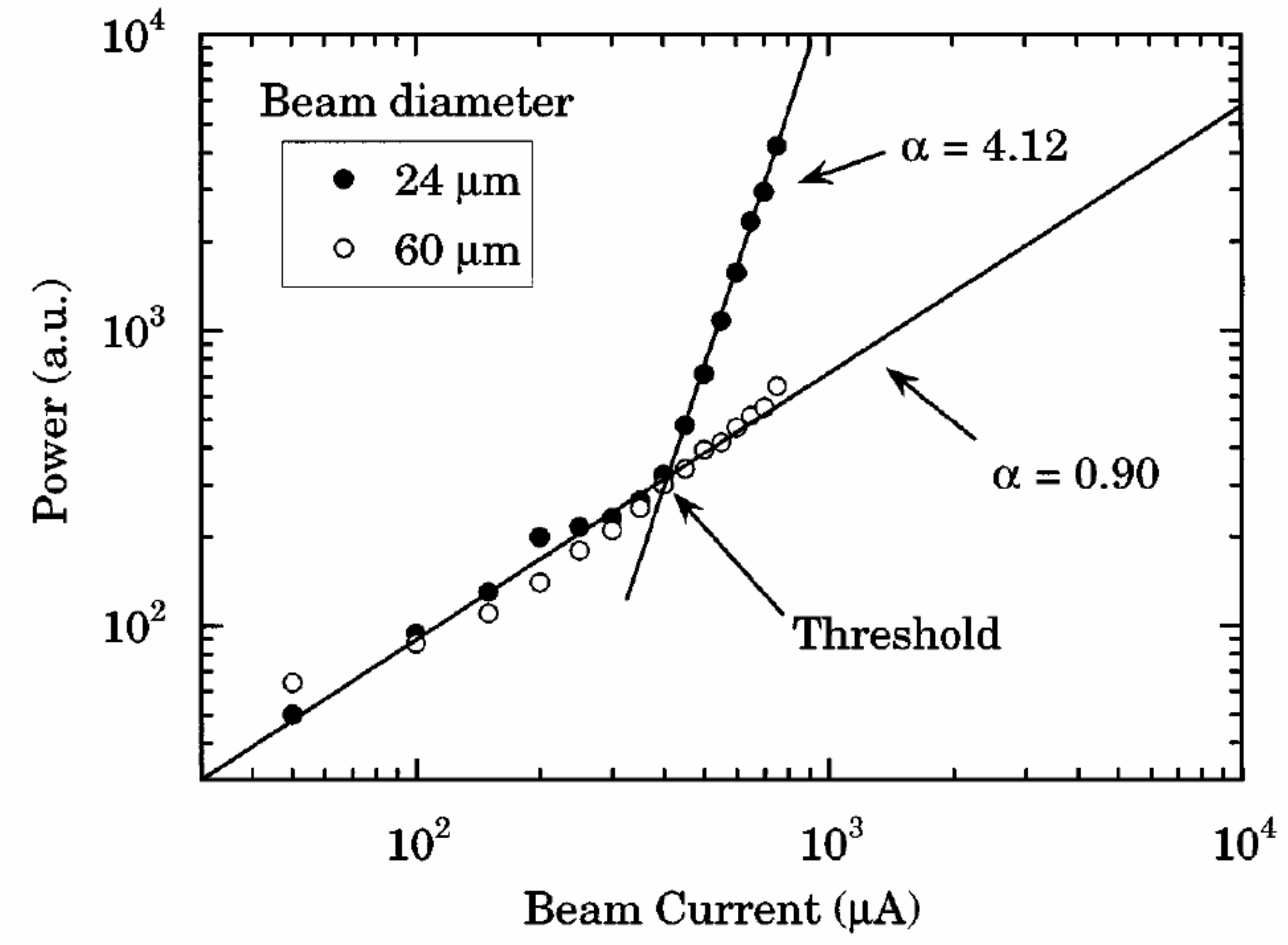
θ	λ
0	321 μm
$\pi/2$	494 μm
π	667 μm

$\lambda_{SP} < 667\ \mu m$

*S. J. Smith and E. M. Purcell, Phys. Rev. 92, 1069 (1953)

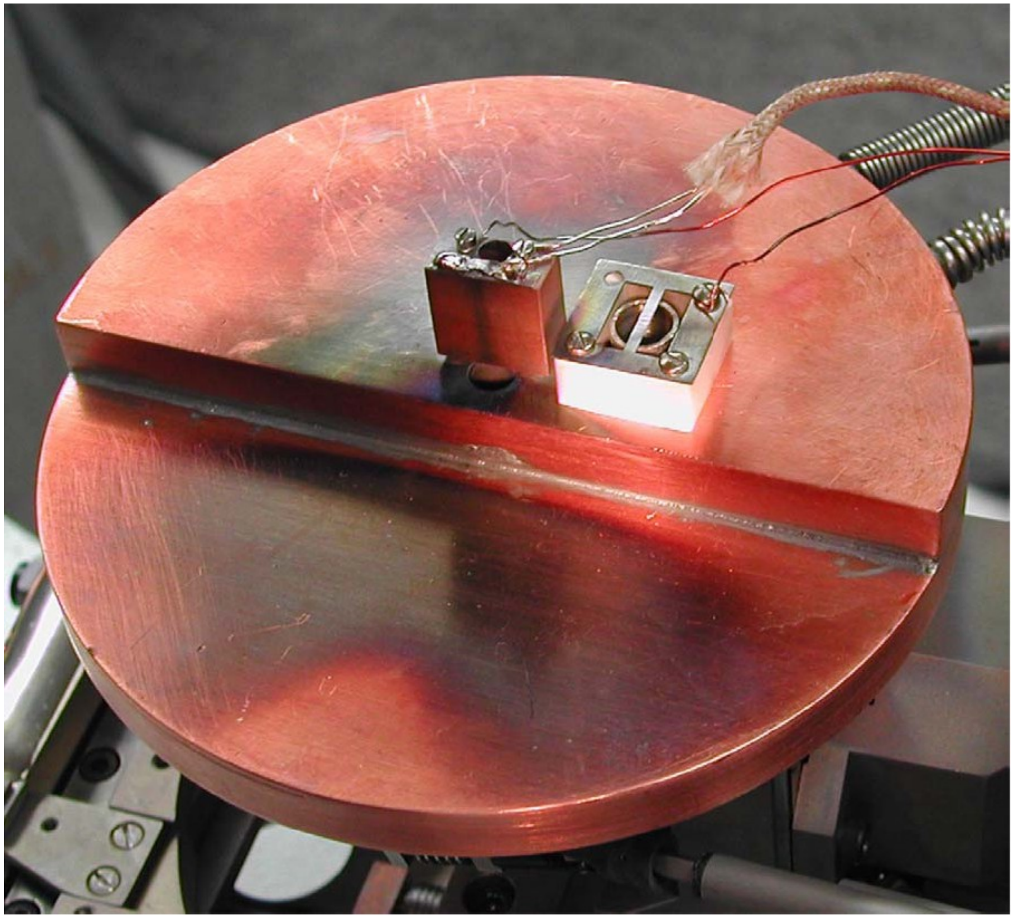
**J. Urata et al., Phys. Rev. Lett., 80, 516 (1998)

Non-Linear Behavior in Smith-Purcell Radiation ? *(Dartmouth, PRL 80 (1998) 516-519)*



Smith-Purcell Experiment using SEM at U of C, Copying the Dartmouth Set-Up (O. Kap, A. Crew, KJK)

Heated Specimen Stage and Possible Black Body radiation background



Smith-Purcell FEL Theory

(V. Kumar and KJK, 2005)

- **Interaction of e-beam with the *surface mode* (freely propagating EM mode with phase velocity βc and frequency $\omega(k_z)$ = c)**

$$\frac{\omega}{k_z} = \beta c$$

- $\frac{d\omega}{dk_z} < 0$: Group velocity in the opposite direction (C. Brau)

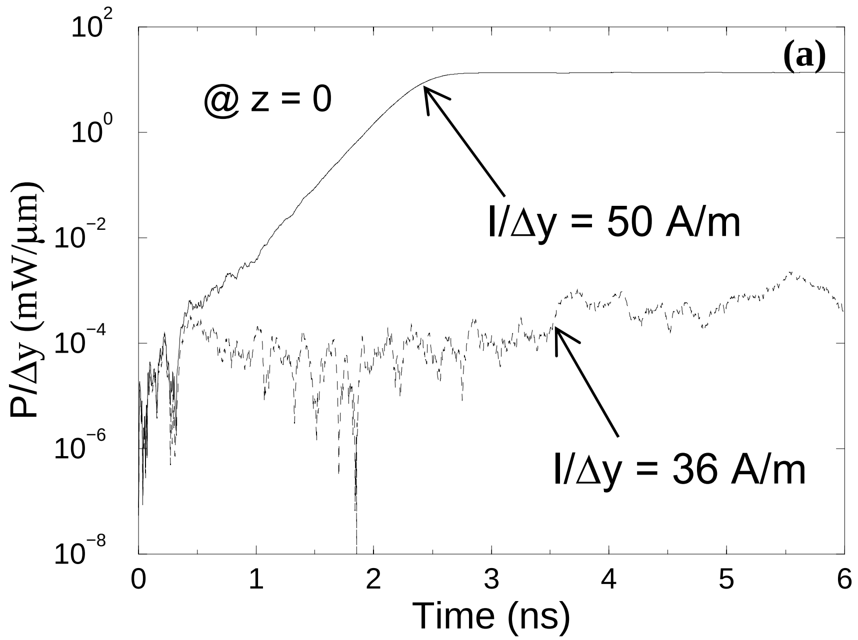
- ***Thus SPFEL is a Backward Wave Oscillator (BWO)***

  Optical energy accumulates exponentially to saturation *without feedback mirrors*

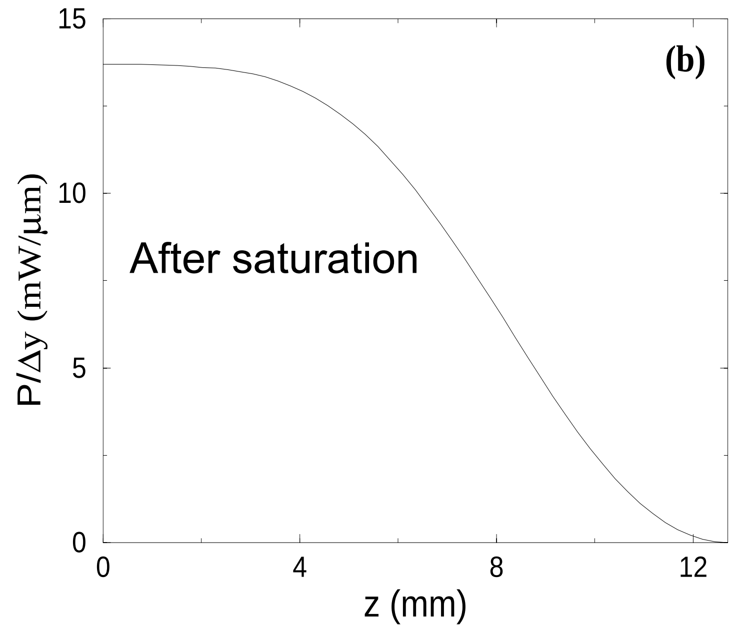
- **Start current condition:**

$$\frac{dI}{dy} \geq \frac{dI_{sat}}{dy} = 7.7\, I_A \frac{(\beta\gamma)^4 \lambda}{(2\pi)^2 \chi L^3} e^{\Gamma_o b}$$

Simulation Results



Energy conversion efficiency = 0.8%



For $I/\Delta y$ = 50 A/m, at saturation, $P/\Delta y$ = 13.7 mW/μm

Power e-folding time = 0.2 ns (simulation)

0.17 ns (analytic formula)

Lasing wavelength = 694.5 μm (simulation)

694 μm (analytic formula)

Smith-Purcell FEL for Terahertz Radiation Requires Flat Beams *(KJK & V. Kumar)*

- **Beam distance to grating surface should be** $\lesssim \beta\lambda/4\pi$

$$b \lesssim 20\mu$$

- **Beam width should be similar to diffraction limit**

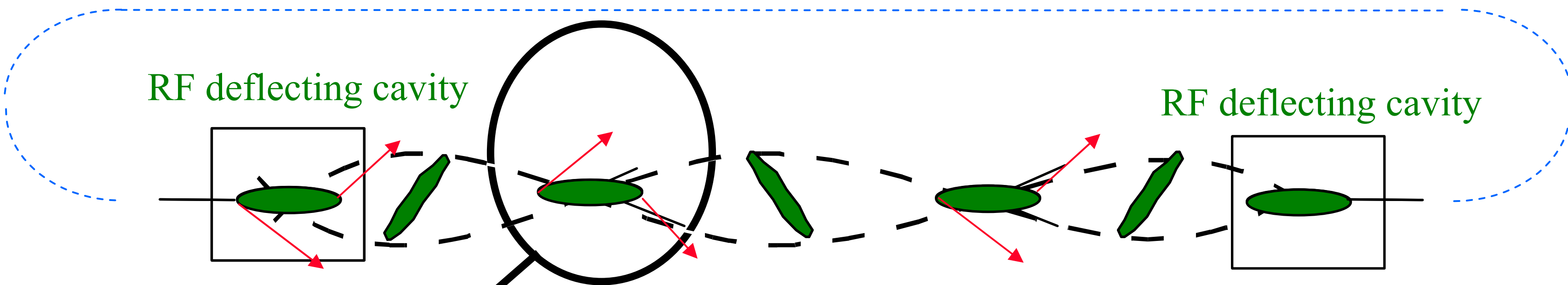
$$\Delta y \lesssim \sqrt{\frac{\beta\lambda L}{4\pi}} \sim 500\ \mu$$

- **A set of beam parameters has been worked out satisfying the transverse profile, start current limit, and the requirement that the space charge effect in beam transport**

- **Miniature Terahertz source**

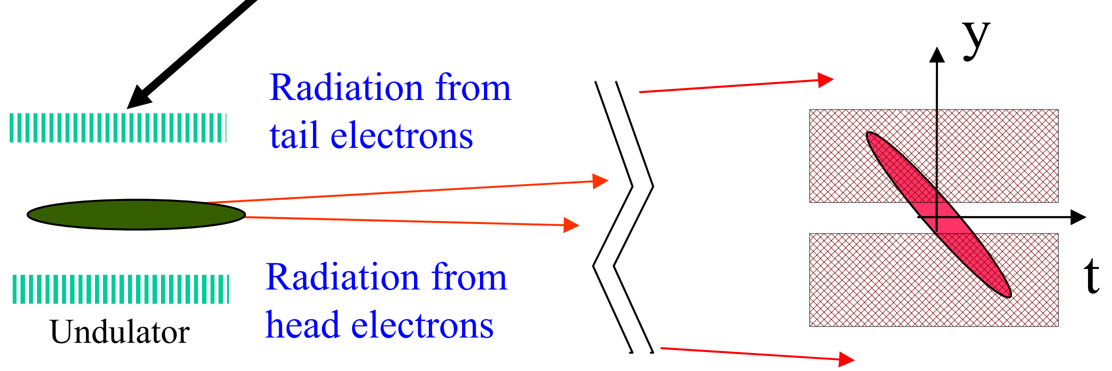
Pulse Compression via Transverse-Longitudinal Correlation (A. Zholents,..)

(Adapted from A. Zholents' August 30, 2004 presentation at APS Strategic Planning Meeting.)



Cavity frequency is harmonic *h* of ring rf frequency

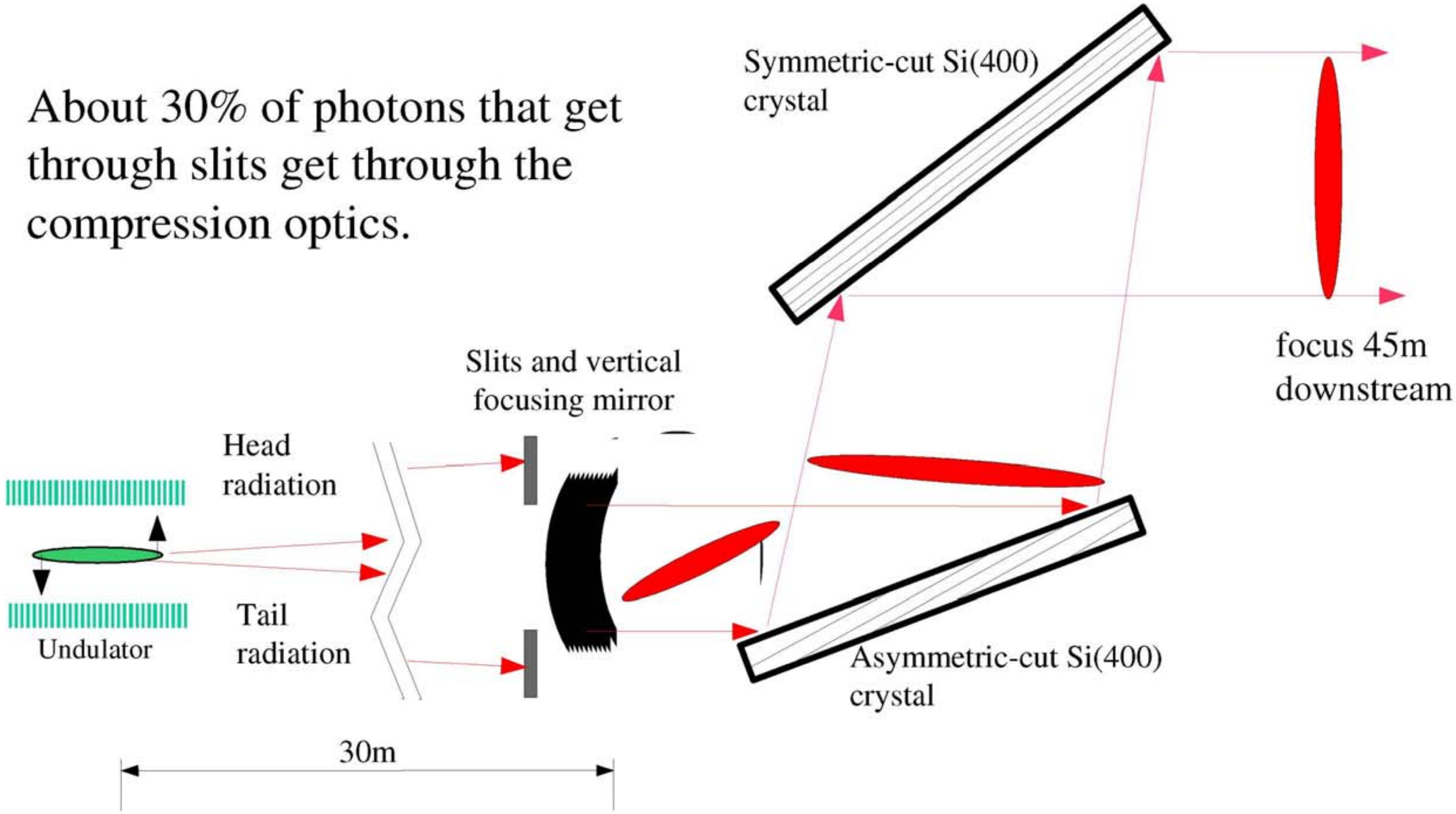
Ideally, second cavity exactly cancels effect of first if phase advance is n*180 degrees



Pulse can be sliced or compressed with asymmetric cut crystal

Preliminary Optics Concept for 10 keV

About 30% of photons that get through slits get through the compression optics.



N.B.: Sketch not to scale. Angles are exaggerated.

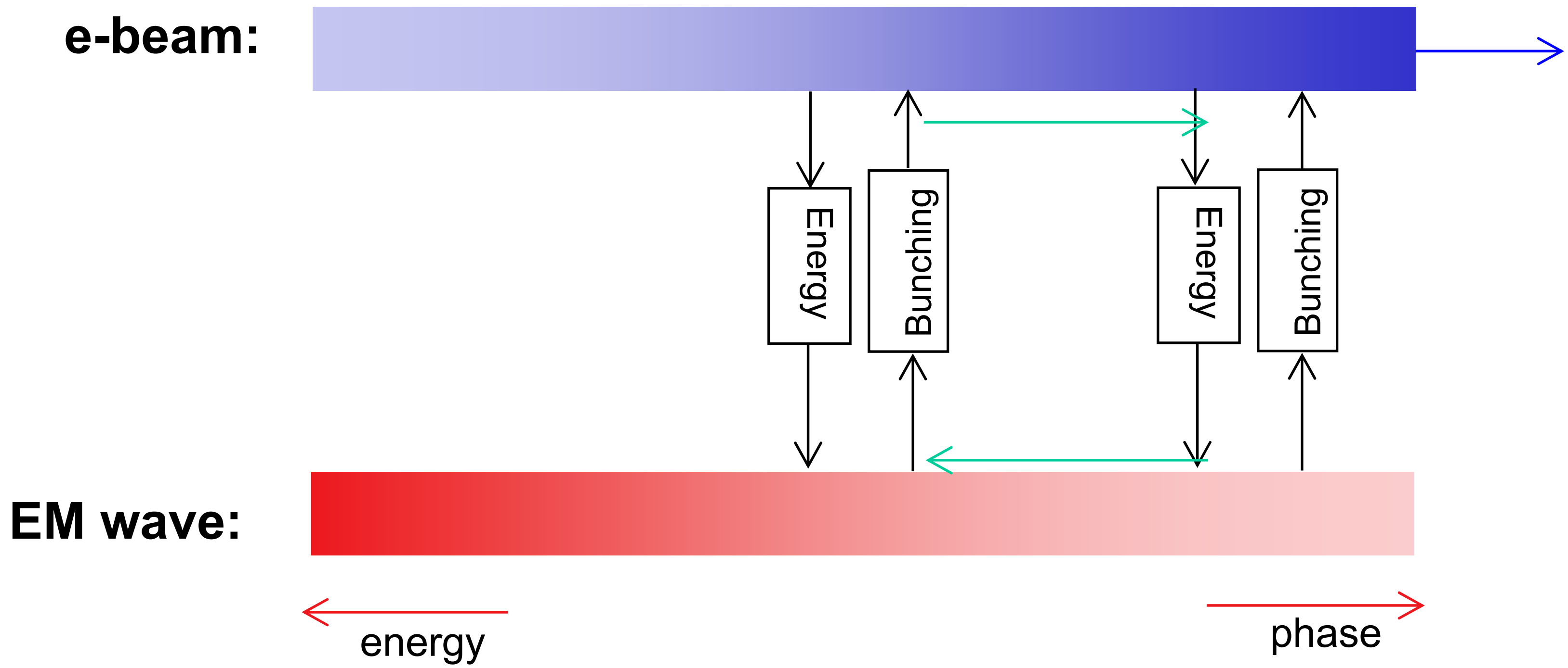
After S. Shastri, APS

X-Ray Pulse Compression at the APS

- ERL with flat beam can achieve femtosecond x-rays (LUX @ LBL)
- A modest but significant compression of the x-ray pulses from 100 ps → 1 ps can be achieved at the current APS setting by installing deflection a pair of cavity
- Together with the advantage of operating user facility
  → Enthusiastic support from APS users

The End

Smith-Purcell BWO

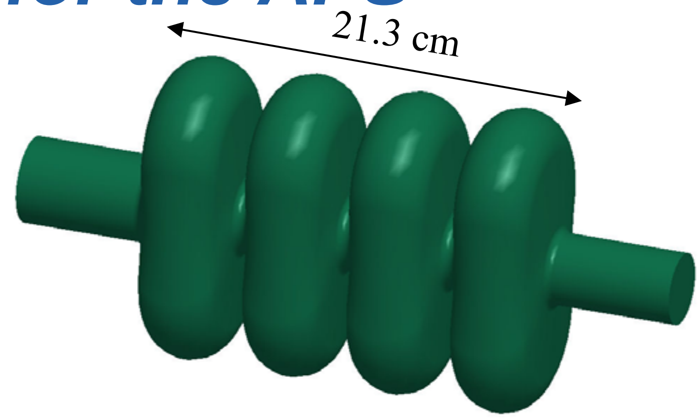


- **At 690 μm, owing to the negative group velocity of the surface mode, there exists a feedback mechanism even without external resonator. → Backward Wave Oscillator (BWO).**
- **Hence, the e-beam has strongest interaction around 690 μm.**

Squashed SCRF Cavities for the APS

- Squashed-cell shape was used to remove TM_{110} degeneracy.
- Modeled after KEK design (cell aspect ratio ~ 1.8).
- Pi-mode chosen for 4-cell cavity to minimize number of cells. Other modes have better frequency separation.
- 4-cell cavity has 230 mT maximum magnetic field. To ensure B_{MAX} < 100 mT, three 4-cell cavities would be required.
- 1-cell cavity has 665 mT maximum magnetic field and would require seven cavities.
- 4-cell cavity has better R_T/Q and will be much more compact than 1-cell cavities.

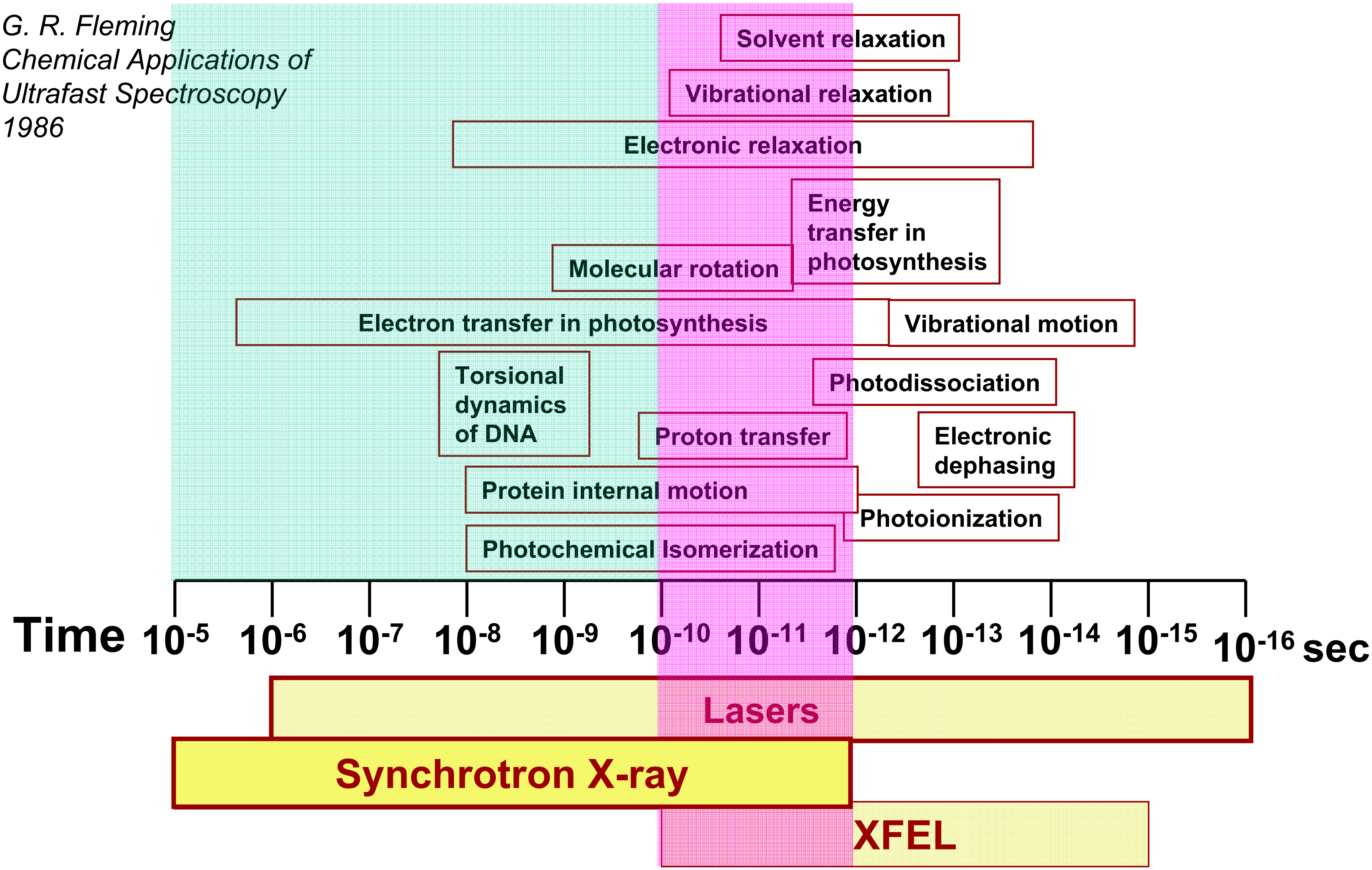
J. Waldschmidt



	1-cell	4-cell
Frequency	2.81 GHz	2.81GHz
λ	10.6 cm	10.6 cm
V_T	4 MV	4 MV
Active Cavity Length	5.3 cm	21.3 cm
R_T/Q	53 Ω/m	230 Ω/m
Q	3×10^9	3×10^9
P_L	102 W	25 W
B_{MAX}	665 mT	230 mT
Cell aspect ratio	1.8	1.8

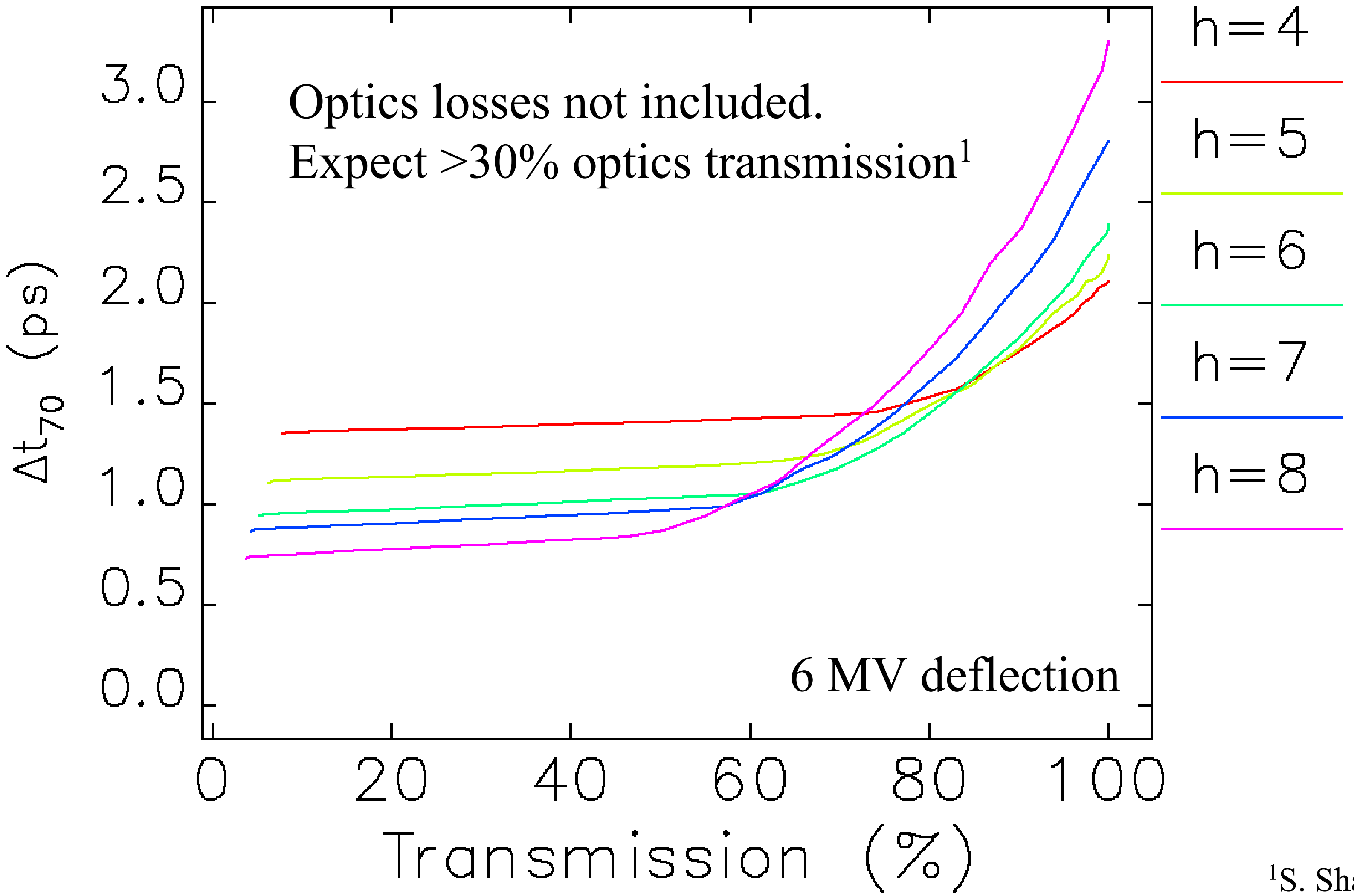
Multiple Temporal Scales in Chemical Sciences

G. R. Fleming
Chemical Applications of Ultrafast Spectroscopy
1986



Compression Results for 10 keV, UA

M. Borland



[1]S. Shastri

Minimum Achievable Pulse Length

(M. Borland)

Electron beam energy

$$\sigma_{t,xray} = \frac{E}{Vh\omega_a}\sqrt{\sigma^2_{y',e} + \sigma^2_{y',rad}}$$

Deflecting rf voltage & frequency

Unchirped e-beam divergence (typ. 2~3 μ-rad)

Divergence due to undulator (typ. ~5 μ-rad)

→ For 6 MV, 2800MHz (h=8) deflecting system, get ~0.4 ps!

- ***Normal APS bunch is 40 ps rms***

Emittance Requirements for X-Ray FELs

- **Requirements for x-ray FELs**

  $\gamma\,\varepsilon_x, \gamma\,\varepsilon_y \;\lesssim\; \mathbf{0.1 \times 10^{-6}}$ **m-rad**

  $$\frac{\delta\gamma}{\gamma} \lesssim 10^{-4}$$

- **However, current state-of-the-art:**

  $\gamma\,\varepsilon_x \sim 1 \times 10^{-6}$ m-rad

  $\delta\,\gamma/\gamma \lesssim 10^{-6}$ ($mc^2\,\delta\gamma = 2.5$ keV)

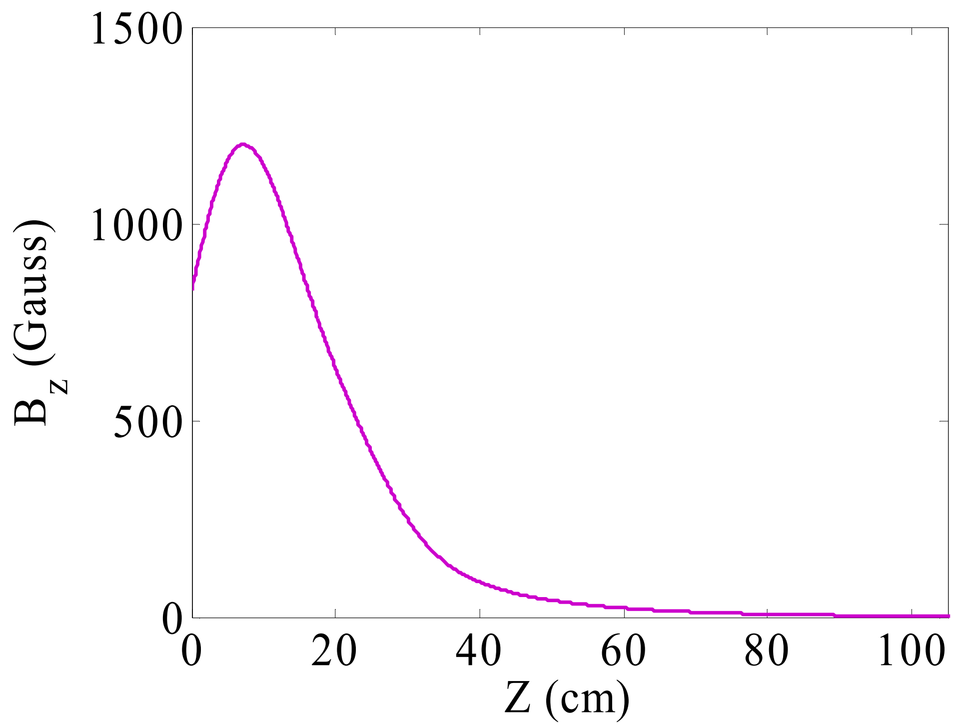
- **Can phase space areas be exchanged?**

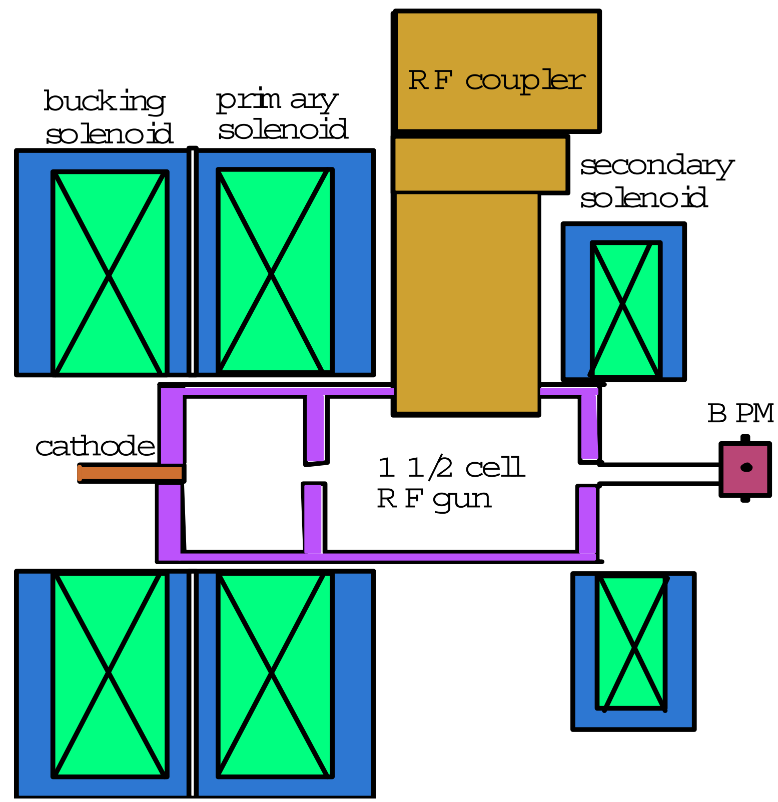
  $$\gamma^2 \varepsilon_x \varepsilon_y \otimes \frac{\delta\gamma}{\gamma} = \left(10^{-6}\ \mathrm{m-mrad}\right)^2 \otimes 10^{-6} \rightarrow \left(10^{-7}\ \mathrm{m-mrad}\right)^2 \otimes 10^{-4} \quad ?$$

Production of Angular Momentum Dominated Beam

$$L = \gamma m r^2 \dot{\phi} + \frac{1}{2} e B_z r^2$$

On the photocathode: $\langle L \rangle = e B_0 \sigma_c^2$





FNPL 1.625-cell RF gun, 1.3 GHz