27th March 2000

CERN Higgs searches: CL_s

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RAL

Talk overview

- **Definition of CL_s**
- **Application in Higgs search**
- **What about Discovery?**
- **Nuisance Parameters**
- **Techniques for calculation**
- **Conclusion**

Requirements of a CL

- **Initially seen from a frequentist perspective**
- **Modified by Bayesian interpretation**

- **Need to be acceptable to community So must satisfy BOTH schools**

Nb Powerpoint thinks both frequentist and Bayesian are spelt wrong..

Definition of CL_b and CL_{sb}

- **Frequentist Definition:**

$$CL_b \equiv P(L \leq L_{obs})$$

Background ensemble

$$CL_{sb} \equiv P(\mathrm{L} \leq L_{obs})$$

Signal +Back ensemble

- 2 hypotheses considered, and only two!
- Ordering automatically 1 sided (in likelihood)

Definition of CL_s

- **CL_s is a safer CL_{sb}**

$$CL_s \equiv CL_{sb} / CL_b$$

- **Used only to Exclude a signal**
  - **CL_{sb} was frequentist CL, CL_s is *LARGER* so conservative - Frequentist-safe**
  - **Asks `How much more unlikely from s than b?' like LR:**

$$\frac{L(s+b)}{L(b)} \approx \frac{CL_{sb}}{CL_b}$$

- Bayes-like

Definition of CL_b'

- **CL_b' is a safer CL_b**

$$1-CL_{b'} \equiv (1-CL_{sb})/(1-CL_b)$$

- **Used only to Discover a signal**
  - **CL_b was frequentist CL, CL_b' is *SMALLER* so conservative** **- Frequentist-safe**
  - **Asks `How much more unlikely from b than s?' like LR:**

$$\frac{L(b)}{L(s+b)} \approx \frac{1-CL_{sb}}{1-CL_b}$$

- Bayes-like

CLRC

Typical PDF distribution:

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Low ll:
Exclusion

medium ll:
no conclusion

high ll:
Discovery

Cl_s always increase by construction

CL_s and

CL_{sb} similar

Clear PDF distribution:

If separation was much larger we would not use statistics

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Treatment of results outside either remains a potential problem!

CLs and

CLsb identical

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CL_s and CL_{sb} distinctly different

Useless PDF distribution:

This is the case where CL_{sb} feels wrong

Rev. Bayes!

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CL_{sb} allows exclusion. CL_s does not.

Clear Poisson Distribution:

3 events observed

Signal of 10 excluded

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CLRC

Typical Poisson Distribution:

Signal of 4

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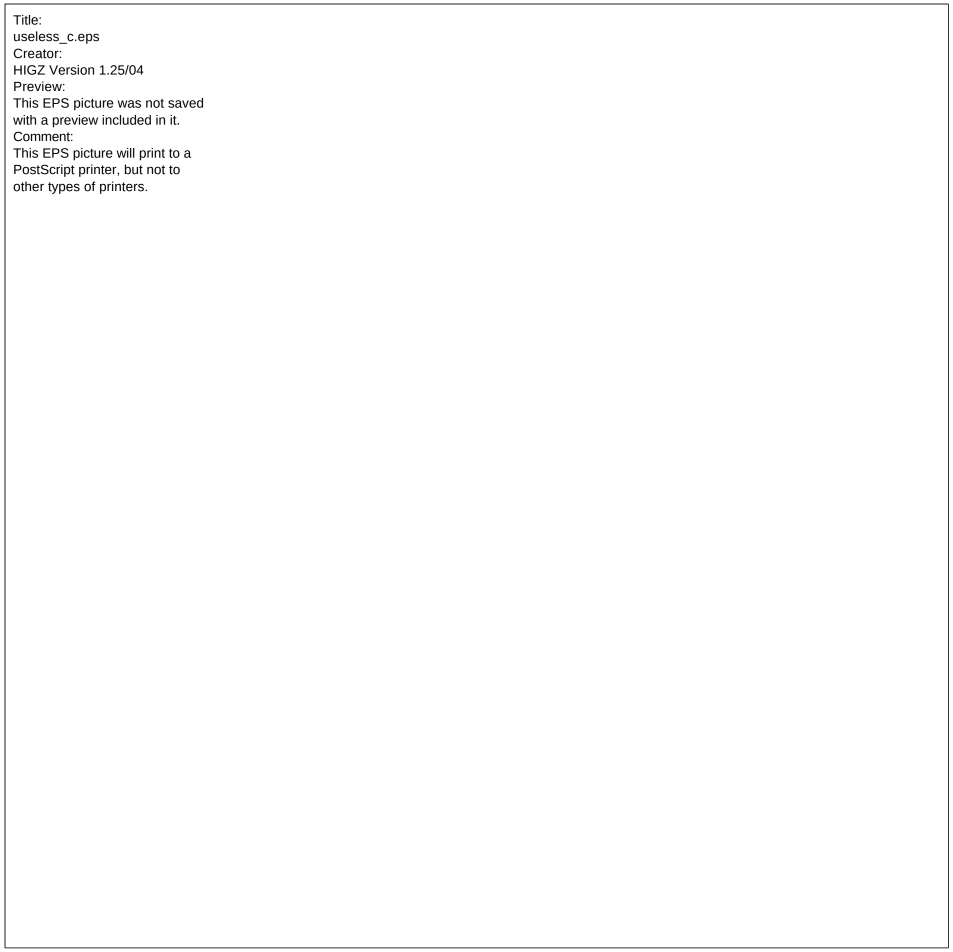
For 3 seen, CL_s is always ~twice CL_{sb}

Useless Poisson Distribution:



Nb:

With CL_s,
0 observed
is always
excluded at

e^{-s}

How did we find this definition?

- **It was an extension of the RPP `96:**

$$1 - e = 1 - \frac{e^{-(m_B+N)} \sum_{n=0}^{n_0} \frac{(m_B + N)^n}{n!}}{e^{-m_B} \sum_{n=0}^{n_0} m_B^n}$$

- **This is the same as CL_s for Poisson**

Why not Feldmann & Cousins?

- **There are drawbacks to F+C:**
  - Limits below e^{-s} when 0 events seen
  - Needs more information than we have! (Some experiments treat each Higgs mass as a separate search, and return `independent' results)
  - Limit can benefit from fluctuations elsewhere
- **It has advantages**
  - Solves the look-elsewhere
- **Not clear whether automatic 2-sided limits are an advantage**

Summary of CL_s

- **Gives overcoverage for classical limits**
- **Outperforms the Bayesian integral with a flat prior in signal rate**
- **Deontologically acceptable - i.e. does not exclude where no discrimination**

(C) P. Janot

- **Does not just tell us whether it is raining**

Application in Higgs search

- **How powerful are the techniques?**

- **Higgs rate v mass (Autumn 99 LEP)** →

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Observed Confidence Levels

LR is same as R value proposed by d'Agostini

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CL_{sb} and CL_s converge for low masses due to fluctuation

Expected Confidence Levels

What does expected mean?

•**Mean**

Has normally been used by us.

•**Median**

No dependence on metric

Careful: Both are used here!

Expected limits:

CL_s .3GeV below CL_{sb}

LR: 1GeV below CL_{sb}

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Probability of Exclusion

Define exclusion as CLs<0.05

Probability of false exclusion should be 5% - but is less

Significant overcoverage

False exclusion rate is always 5% *of the true exclusion rate*

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What about Discovery?

- **CL_b is the accepted indicator (CL_b' under study)**
- **Require 5σ => $1-CL_b < 5*10^{-7}$**
- **No real allowance for flip-flopping**
  - **Can (*will*!) ALWAYS quote limit**
  - **flip-flop probability VERY small**

Look-elsewhere effect

- **Can discover at ANY mass - raises probability of fake discovery above $5*10^{-7}$**
- **Results are each mass are correlated**
- ***But:* What mass range should be checked?**
  - **Last years limit to sensitivity limit?**
  - **Full range scanned? (But that is arbitrary!)**
- **No RIGHT answer - We use a down-weighting factor, from MC experiments; 4 in SM case.**

Nuisance Parameters

- **See Slides from T. Junk**

Techniques for Calculation

- **Three different methods used:**
  - **Monte Carlo calculation. (A.Read) Flexible but slow. Tricks help.**
  - **Analytic folding (P.Bock)**
    - **event by event: Good for low event nos.**
    - **bin by bin: Good for low bin nos.**
  - **FFT approach: (S.Nielsen) Fastest for large problems**

Monte Carlo calculation

- **Used for current LEP limits**
- **Very easy to add all sorts of complications by varying ensemble**
- **Takes several days CPU for MSSM limits.**

- **Use LR=(s+b)/b to enhance effective statistics**

Conclusions:

- CL_s is well-tested practical solution.
- Safe for Classical statistician
- Bayes-like properties for a Bayesian
- Removes a few hundred MeV w.r.t. to optimal Frequentist CL_{sb}

- No Higgs found yet (m_H>107.7GeV)