Major! Progress in lattice QCD ?

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(HPQCD Collaboration)

Aspen Winter Conference
February 2006

(1) Precision *Unquenched* LQCD

Progress has been made: "Greatest Hits"

Some outstanding theoretical issues

(2) Role of Perturbation Theory

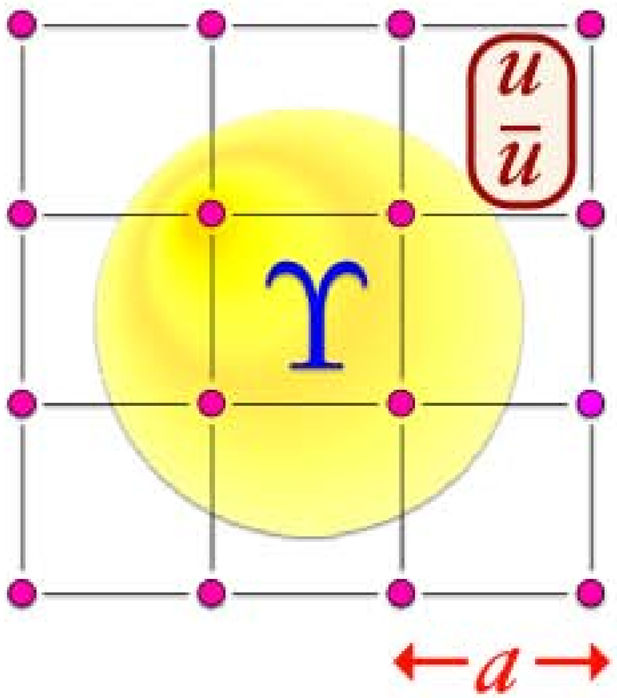
(3) Recent phenomenology

See Ian Shipsey's talk for more theory vs. experiment

(4) Other Approaches

HPQCD, FNAL & MILC

(1) LQCD: *Progress has been made*



For LQCD to be relevant to experiment:

(i) Include the effects of virtual u, d, s quarks: *Unquench*

(and do so at sufficiently small quark masses → physical point)

(ii) Get sufficiently close to continuum limit

(iii) Quantify the statistical & systematic errors

HPQCD & Friends: *Greatest Hits*

$$\langle \mathcal{O} \rangle = \int [dU_\mu(x)][d\bar\psi d\psi]\, \mathcal{O}\, e^{-\beta(S_{\rm gluon} + S_{\rm quark}^{\rm stagg})}$$ ⟸ !

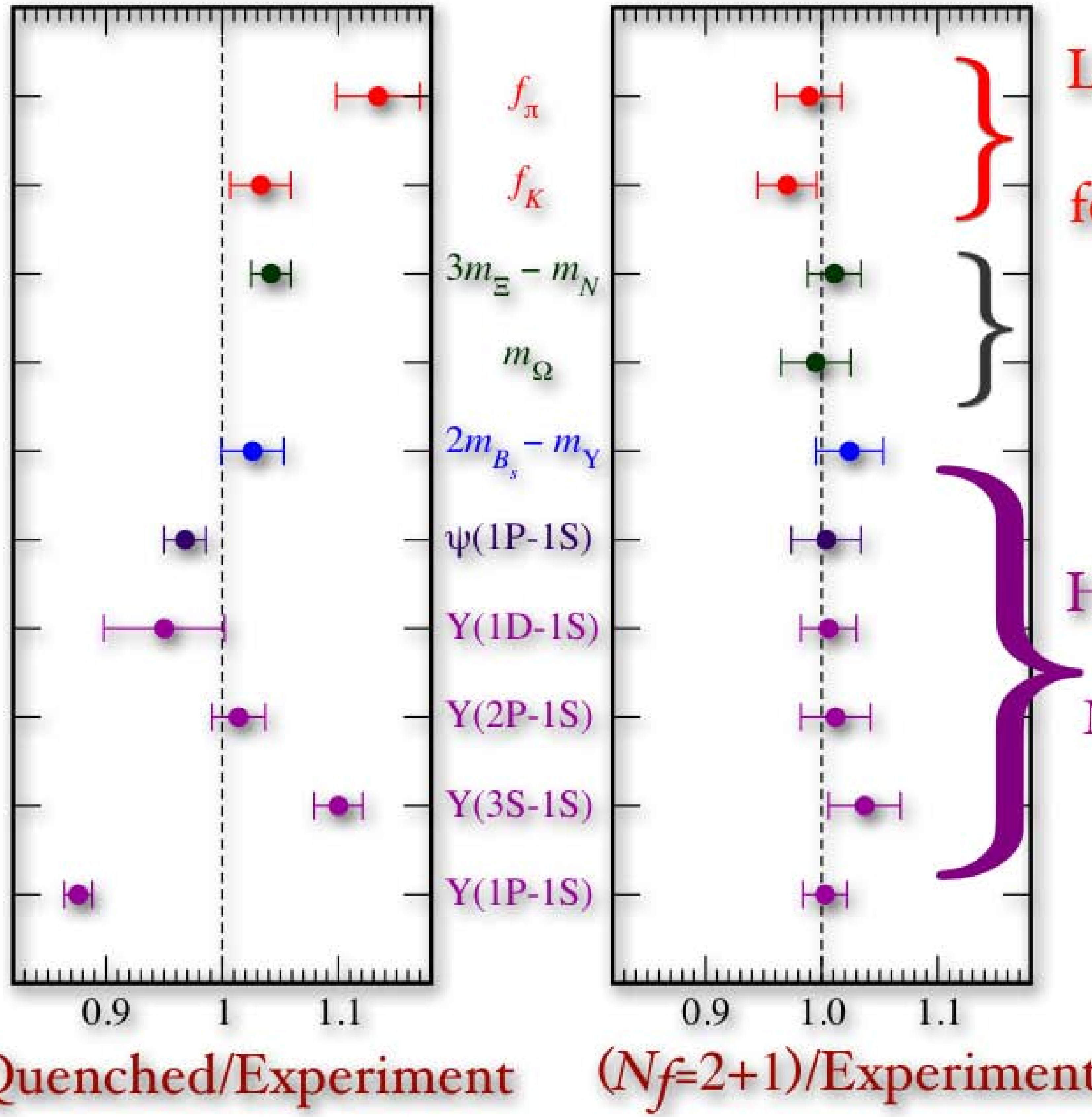
- Only 5 input parameters (same as in continuum QCD)
  - $m_u (= m_d),\ m_s,\ m_c,\ m_b,\ a\ (\Leftrightarrow \alpha_s)$
    - $m_{u/d} \leftarrow m_\pi^2$
    - $m_s \leftarrow 2m_K^2 - m_\pi^2$
    - $m_c \leftarrow m_D$
    - $m_b \leftarrow m_\Upsilon$

    } Each experimental quantity roughly $\propto$ the one $m_{\rm quark}$ and roughly independent of the other masses

    - $a \leftarrow m_{\Upsilon'} - m_\Upsilon$ ⟵ This mass difference roughly independent of all $m_{\rm quark}$'s
- Same ***a*** using *any* input: $f_\pi, m_K, m_{B_s}, \ldots$

LQCD / Experiment

HPQCD, MILC, UKQCD, Fermilab, Collab'ns PRL 92, 2004



Light-quarks & form-factors

Baryons

Heavy quarks:

Note "unified" treatment of e.g. *B* & Υ

Errors ~ 10-15%

Errors < 3%

Partially Quenched χ PT

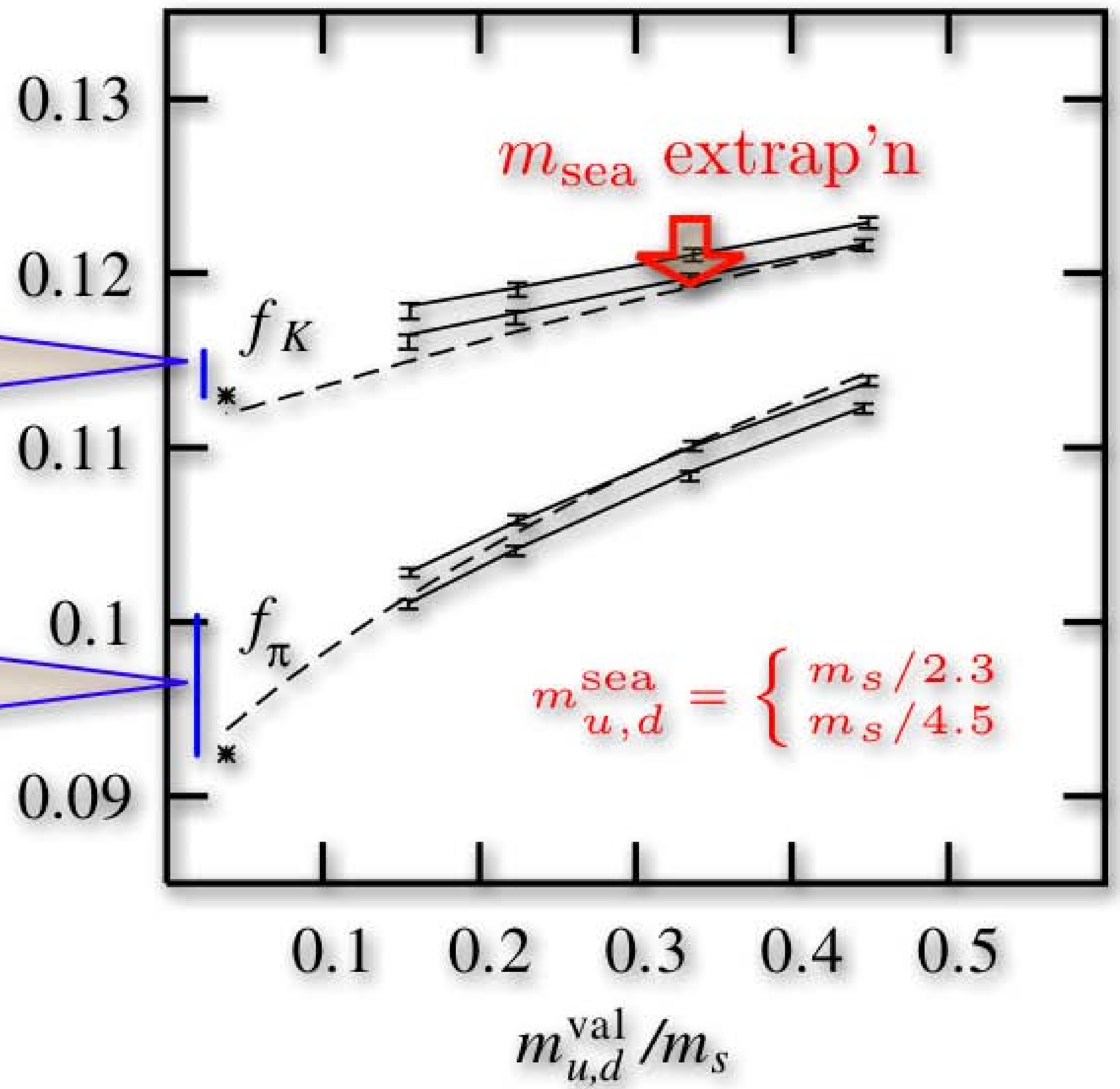
Lee & Sharpe
Bernard & Aubin

Run at several $m_{u/d,s}^{\text{valence}}$ not necessarily equal to m_{sea}

$\approx 4\%$

m_{val} extrap'n

$\approx 9\%$



Sea quark masses ~3-10 times smaller than in previous unquenched

Recent $m_u/m_s \sim 1/10$

LQCD can't do everything (not for now!)

- Unstable hadrons e.g. $\rho \to \pi\pi \to \rho$
  - intermediate particles propagate to lattice boundaries, induces large finite volume errors
- Hadrons near decay thresholds e.g. $\psi' \to D\overline{D} \to \psi'$
- At most one hadron in initial and final states

"Gold-plated" LQCD quantities

- Narrow/Stable hadrons sufficiently below threshold
  - e.g. $\pi, K, p, D, D_s, B, B_s, J/\psi, \Upsilon, \Upsilon', \ldots$ but not $\rho, D^*, \psi', \ldots$
- Semileptonic ✓ while nonleptonic ✘
- CLEO-c offers crucial validation (Ian Shipsey's talk)

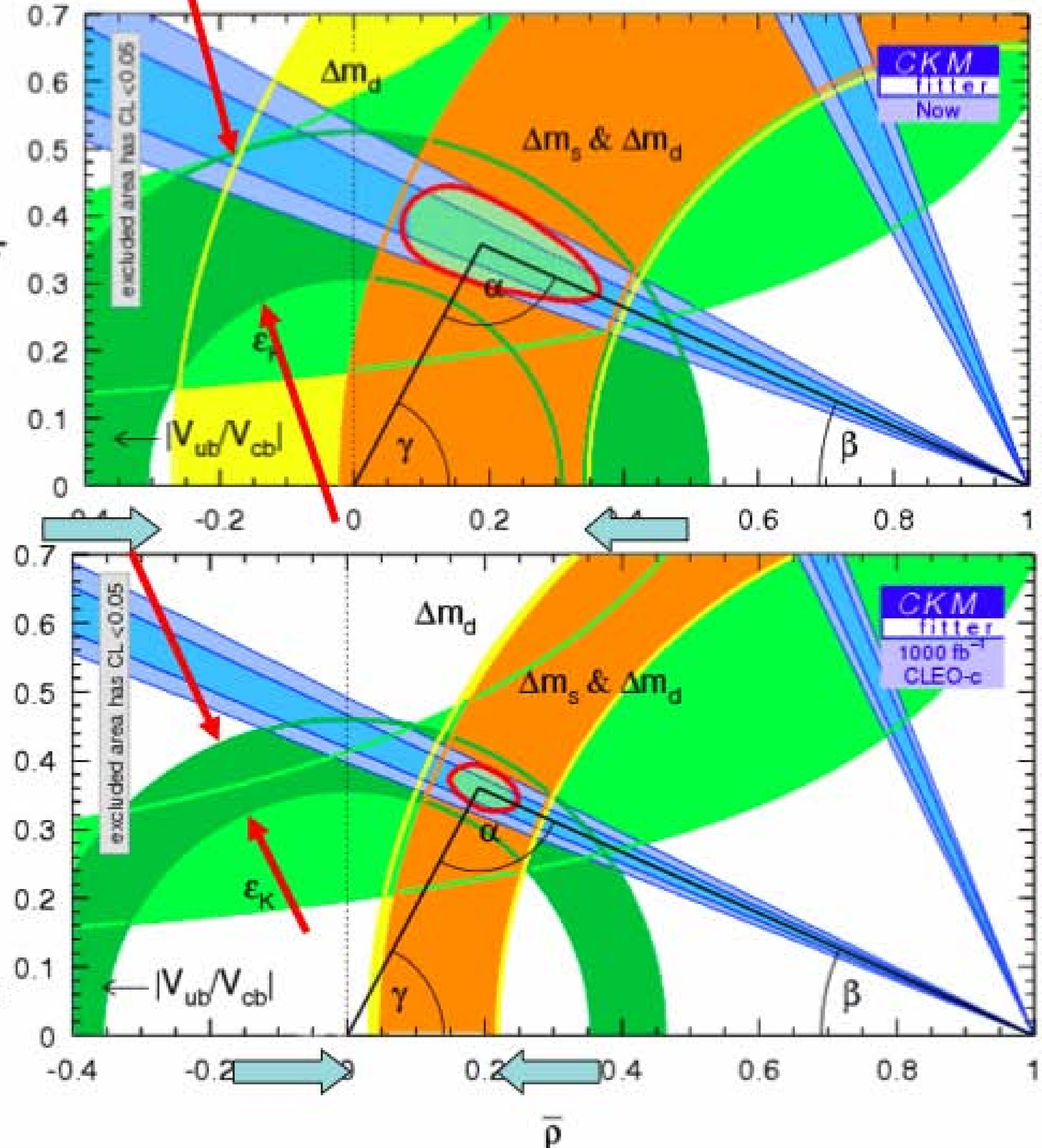
"Gold-plated" LQCD meets CKM

V_{ud}	V_{us}	V_{ub}
$\pi \to \ell\nu$	$K \to \ell\nu$	$B \to \pi\ell\nu$
	$K \to \pi\ell\nu$	
V_{cd}	V_{cs}	V_{cb}
$D \to \ell\nu$	$D_s \to \ell\nu$	$B \to D\ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$	
V_{td}	V_{ts}	V_{tb}
$\langle B_d \vert \overline{B_d} \rangle$	$\langle B_s \vert \overline{B_s} \rangle$	

CKM: Present & Future
(Ian Shipsey)

Precision theory + charm = large impact

Now



Plot uses Vub Vcb from exclusive decays only

Theoretical errors dominate width of bands

precision QCD calculations tested with *precision* charm data at threshold
➔theory errors of a few % on B system decay constants & semileptonic form factors

+

500 fb-1 @ BABAR/Belle

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A potential pit-fall for HPQCD

- Staggered quarks cheap to simulate: remnant χ symmetry
  - Staggered quarks plagued by flavour doubling:

$$\bar{\psi}(x)\,\gamma\cdot D\,\psi(x) \;\Rightarrow\; \sin(p_\mu a)\times\bar{\psi}(p)\gamma_\mu\psi(p)$$

$\Rightarrow$ low-energy modes at $p_\mu = 0, \pi/a$

$\therefore$ 2^4 copies ("tastes") (reduce to 4 tastes by "staggering")

To get desired (2+1)-flavours instead of 4

$$\det(\gamma\cdot D + m) \rightarrow \det(\gamma\cdot D + m)^{1/4}$$

☂ potentially worrisome non-localities

$$\det(\gamma \cdot D + m) \rightarrow \det(\gamma \cdot D + m)^{1/4}$$

Is this a local effective theory ?

☺ Correct to all orders in PT (Batrouni et al 1985)

☺ Chiral anomalies correctly handled (Sharatchandra et al 1981; Smit & Vink 1988)

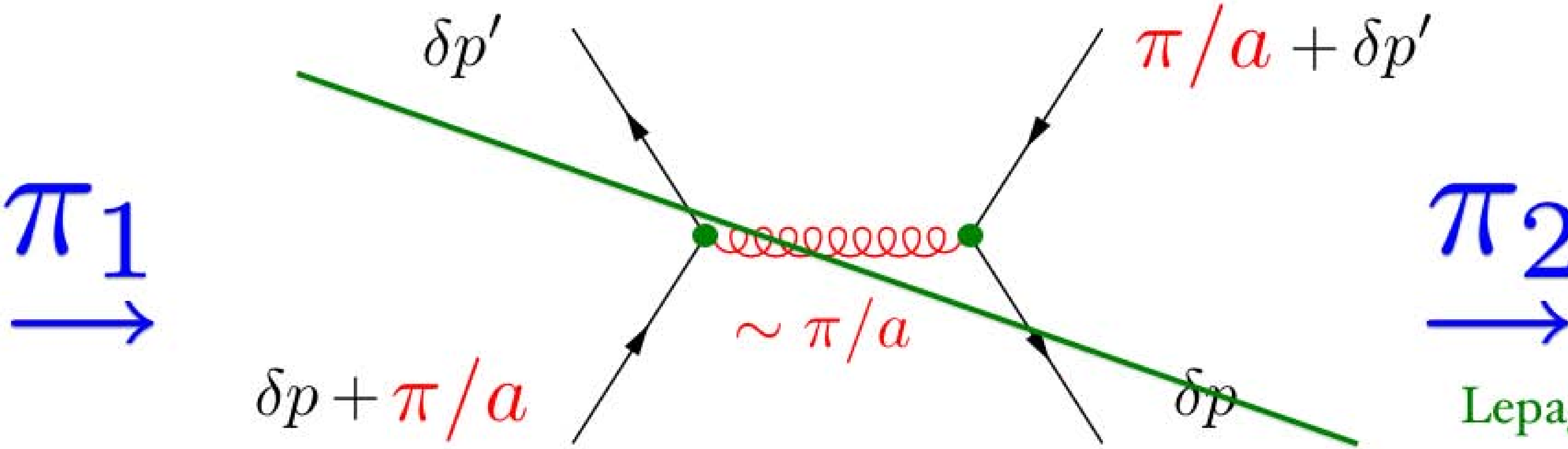
☺ Controlled by short-distance interactions

$$[\det(\gamma \cdot D + m)]^{1/4} = [\Pi_n(\lambda_n + m)]^{1/4}$$

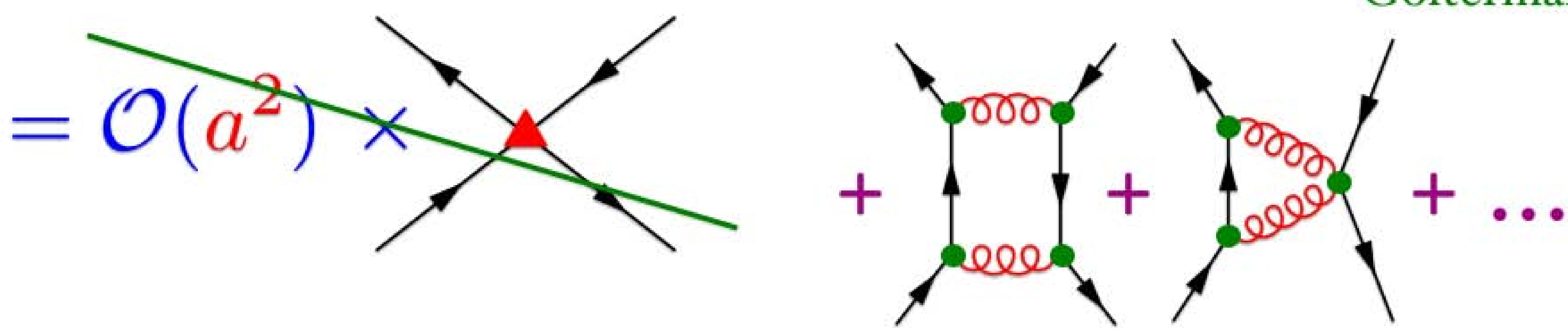
- 1/4 root OK if eigenvalues are ≈ quadruply degenerate
- eigenvalue- or "taste-" splittings controlled by short-distance interactions

.....

Taste-changing interactions



Lepage
Lagae & Sinclair
Golterman



Minimize taste-changing interactions by perturbative analysis of the effective interaction ●

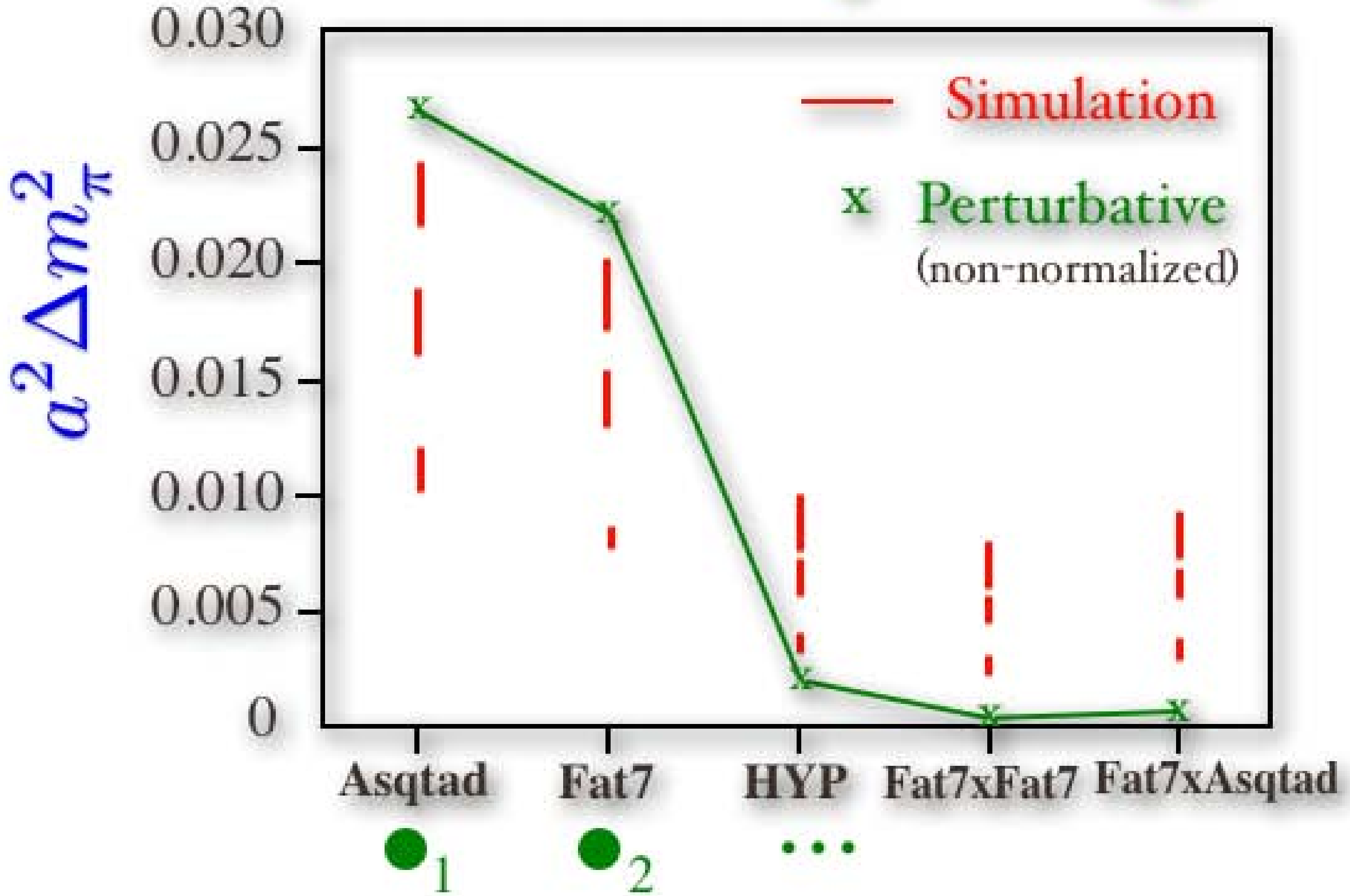
Gave us the "improved" staggered action in current use ... do more!

Pion “taste” splittings

1/4 root

☺

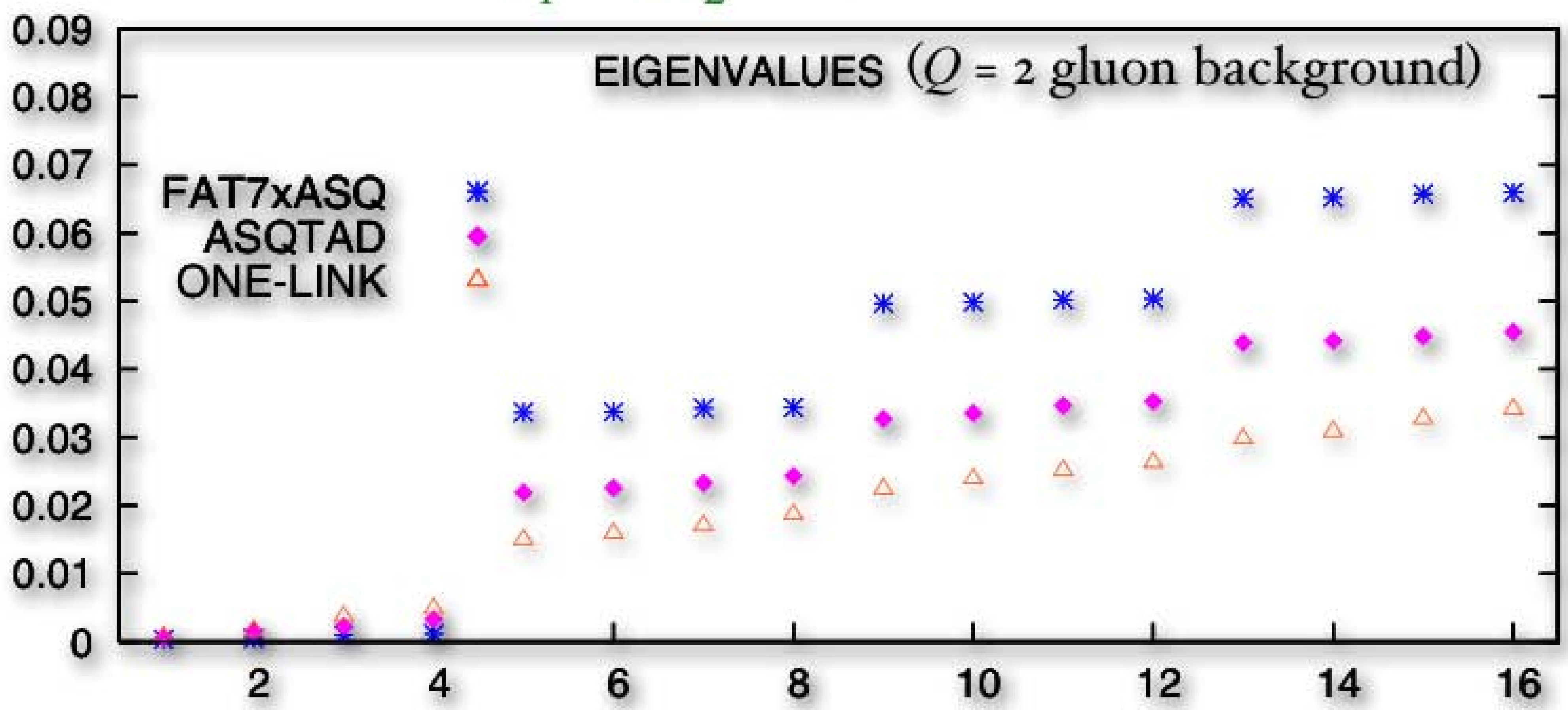
(so far)



Follana, Davies, Hart, Lepage, Mason, *HDT* Lattice 2003

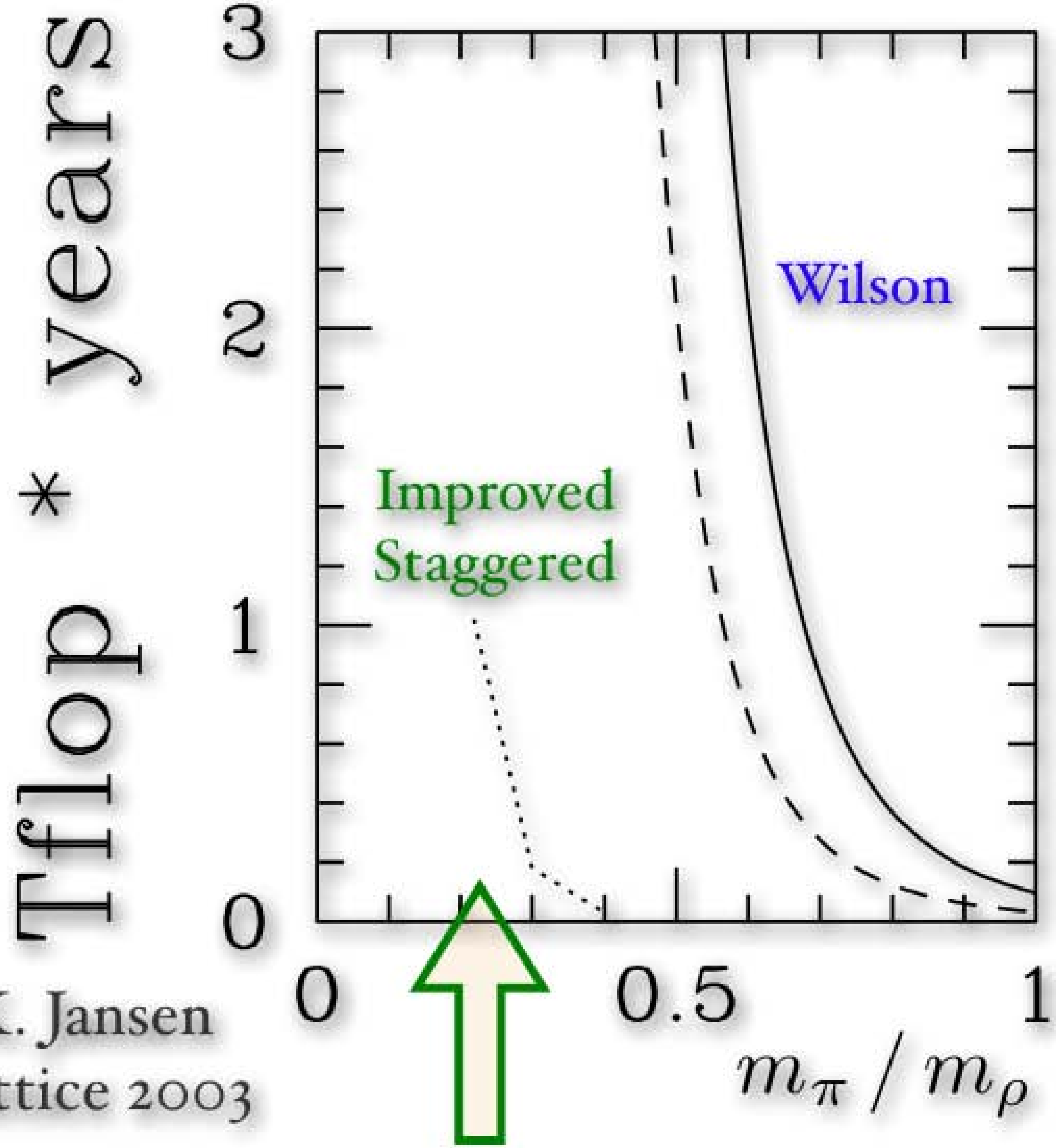
$\not{D}$

Also: Woloshyn & Wong; Durr, Hoelbling, & Wenger



Follana, Hart, Davies, Mason PRL 2004

The price we pay for speed



K. Jansen
Lattice 2003

No surprise: staggered quarks have χ symmetry which prevents zero modes => matrix inversion

Why now?

Discretization effects finally understood using perturbation theory: Lepage & Mackenize

New algorithm Wilson quarks:

Appears to radically lower cost: M. Luscher, Lattice 2005

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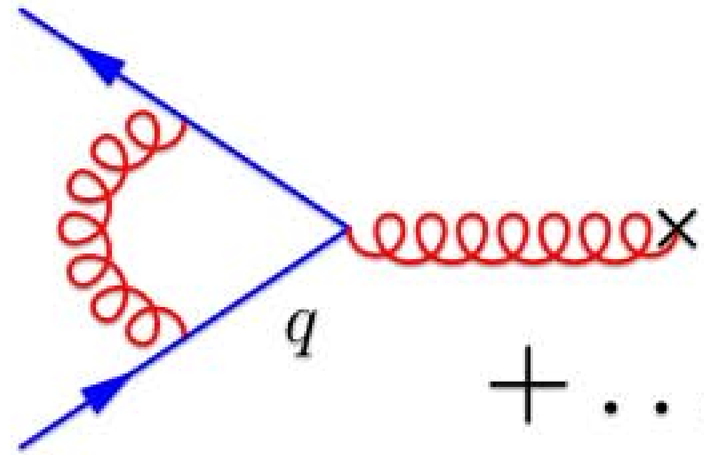
(2) Lattice PT key to HPQCD progress

Why PT when lattice is nonperturbative?

Scattering amplitude $\sim$



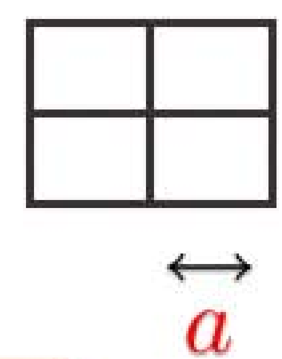
$+ \ldots$

(e.g. off external chromo-B field)

$$\mathrm{Amp}\Big|_{\text{Continuum}} \sim \int_{-\infty}^{\infty} d^4q \ldots$$

How to make up difference?

$$\mathrm{Amp}\Big|_{\text{Lattice}} \sim \int_{-\pi/a}^{\pi/a} d^4q \ldots$$

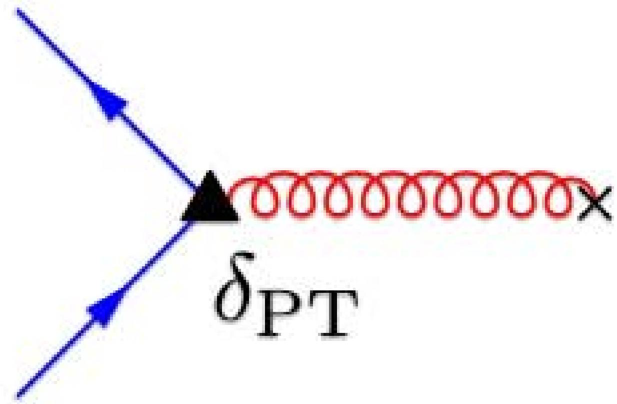


(i) Send lattice spacing $a \to 0$

▶ but lattice simulation cost $\sim 1/a^6$

(ii) Add new contact interactions

$$\mathcal{L}_{\text{lattice}} = \mathcal{L}_{\text{Dirac}} + \delta_{\text{PT}} \bar{\psi} a \vec{\sigma} \cdot \vec{B} \psi + \ldots$$



Perturbative: Fills-in high-energy modes

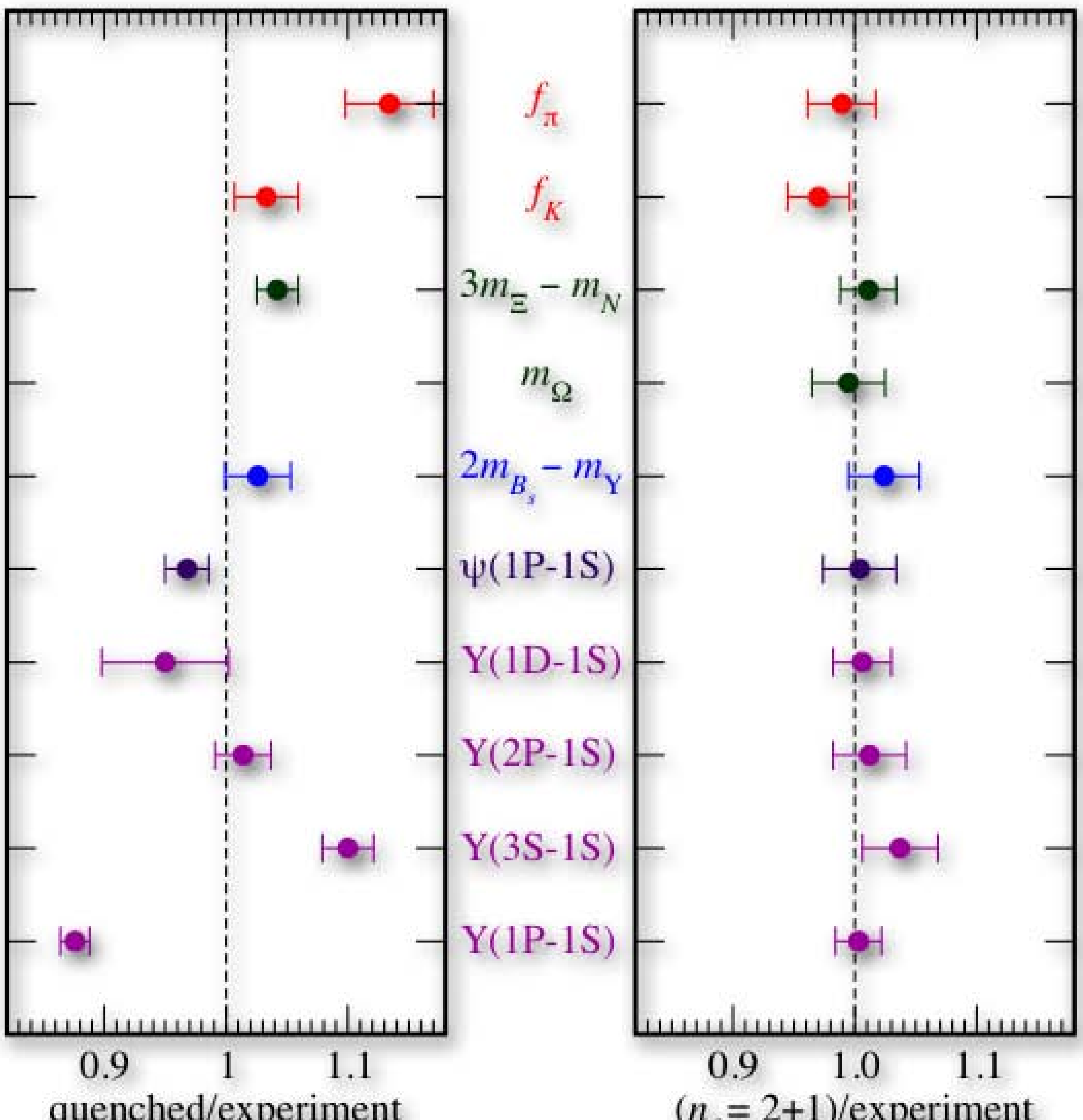
δ_{PT} = "Lattice - Continuum" $= c_1 \alpha_s + c_2 \alpha_s^2 + \ldots$

- Few % precision demands ambitious program:

$$\alpha_s(1/a) \approx 0.2\text{–}0.3 \approx a\Lambda_{\text{QCD}} \quad (a \approx 0.1 \text{ fm})$$

- PT matching through 2nd-order (NNLO) — hard, but in progress!
- $O(a^2)$-accurate discretizations — (HPQCD ✔ but not so all other approaches)

Not much “explicit” PT needed earlier



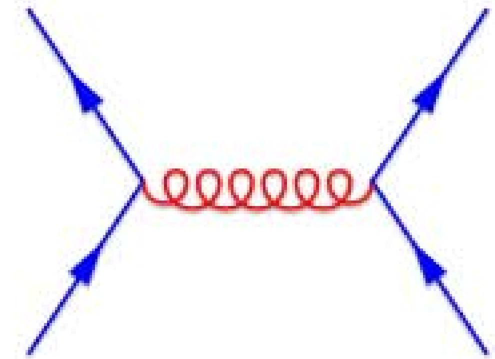
[***Except*** for design of “improved” discretizations] !

- “Absolutely” normalized hadron masses and currents
- However most all other quantities *will* require PT

Lattice regulator makes PT ☹

$$-4\pi C_F \frac{\alpha_V(q^2)}{q^2}$$

=



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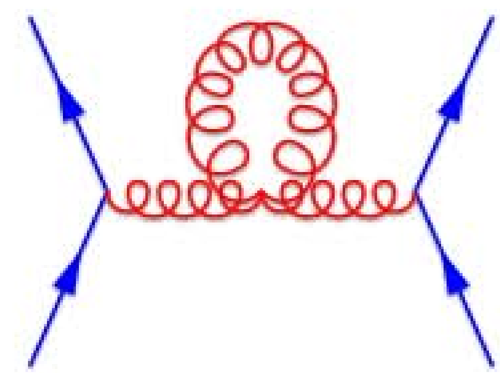

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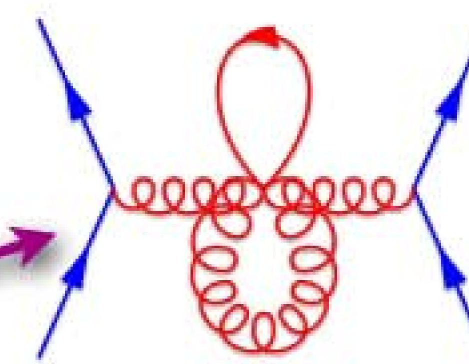
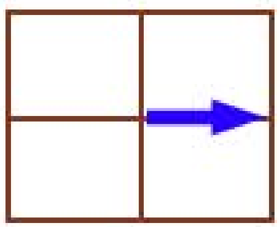


!

10,000's terms

+ ...

Complication is due to link field



$$U_\mu(x) = e^{igaA_\mu(x)}$$

☺ Lattice PT can be automated!

- M. Lüscher and P. Weisz,
  Nucl. Phys. B266, 309 (1986)

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$\alpha_{\overline{\mathrm{MS}}}(M_Z)$ $m_{u,d,s}$ D & B decays

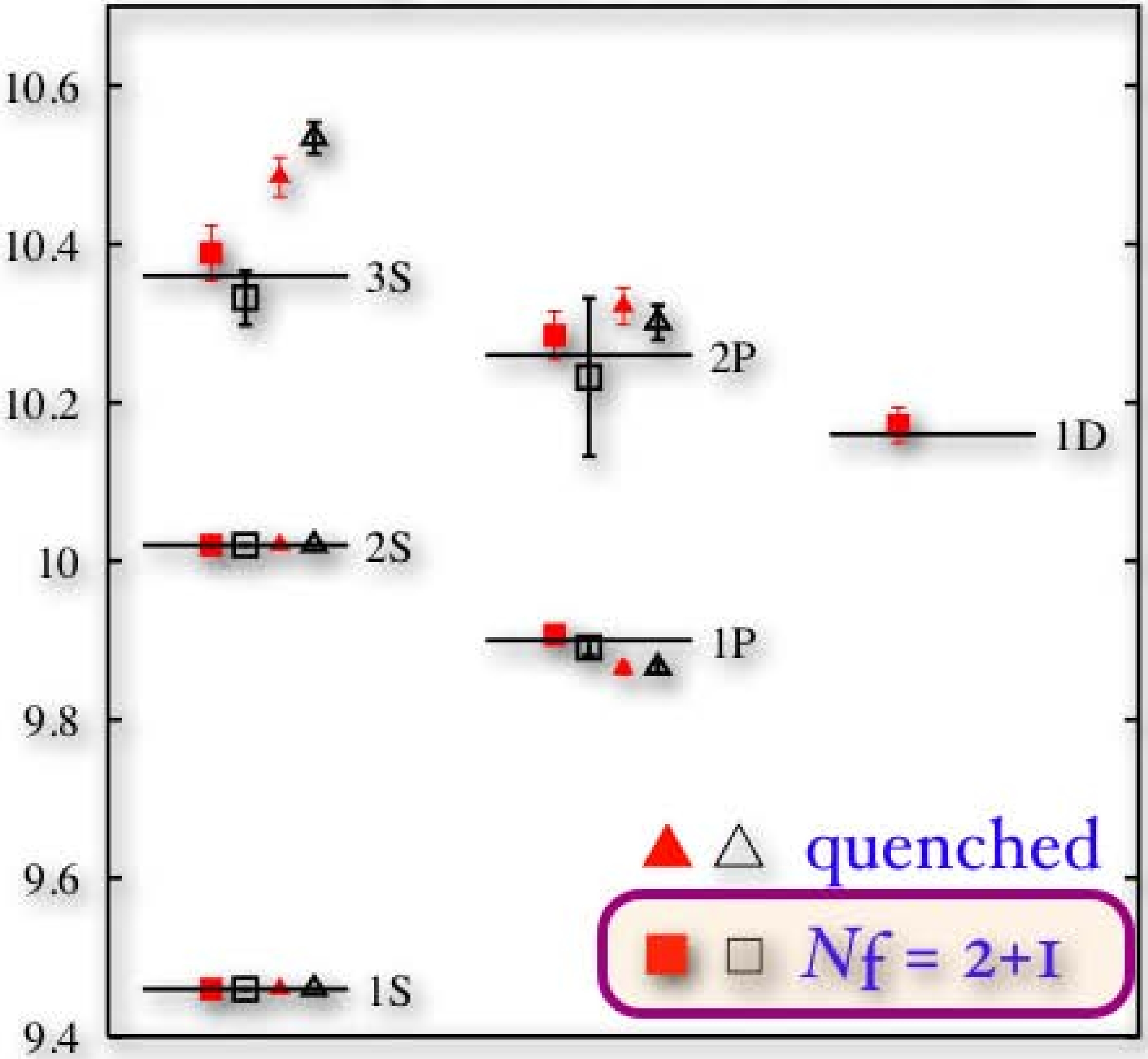
(4) Other Approaches

(3a) Determination of $\alpha_{\overline{\mathrm{MS}}}(M_Z)$

"Novel" use of *both* long- & short distance QCD (NRQCD Collab'n 1997)

Υ spectra (GeV)

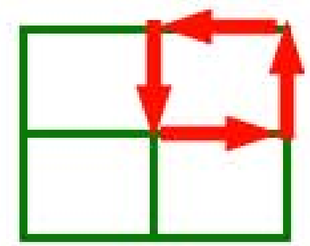
(Gray et al. PRD 72 2005)



What's new?

(i) NPT input e.g. $\Upsilon' - \Upsilon \Rightarrow a$

(ii) Measure short-distance quantity



Wilson loop

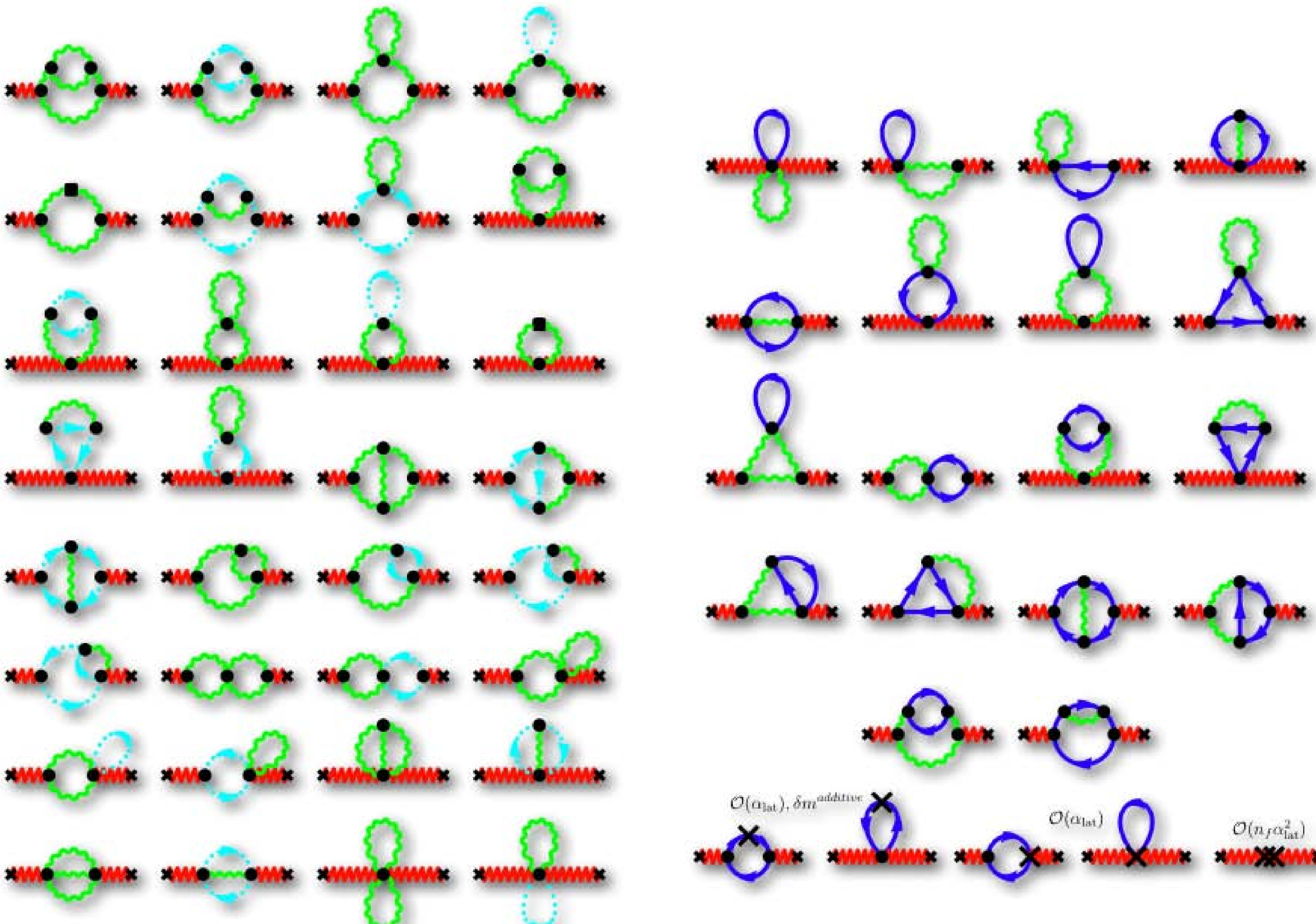
- Characteristic scale $q^* \propto 1/a$

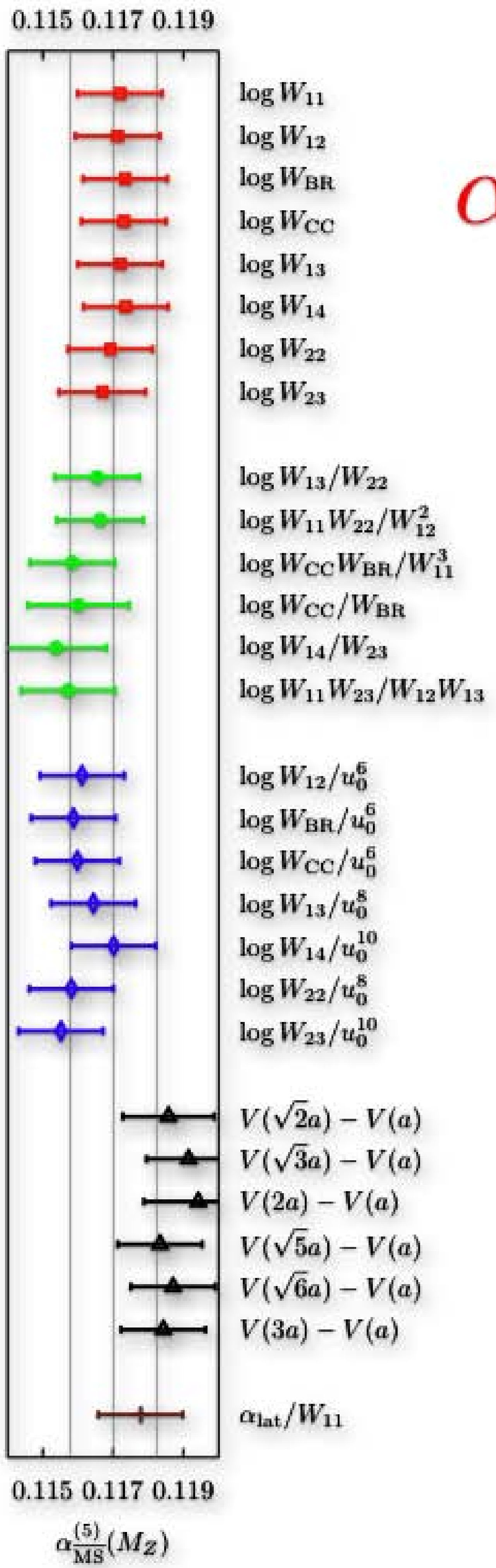
(iii) Use perturbation theory:

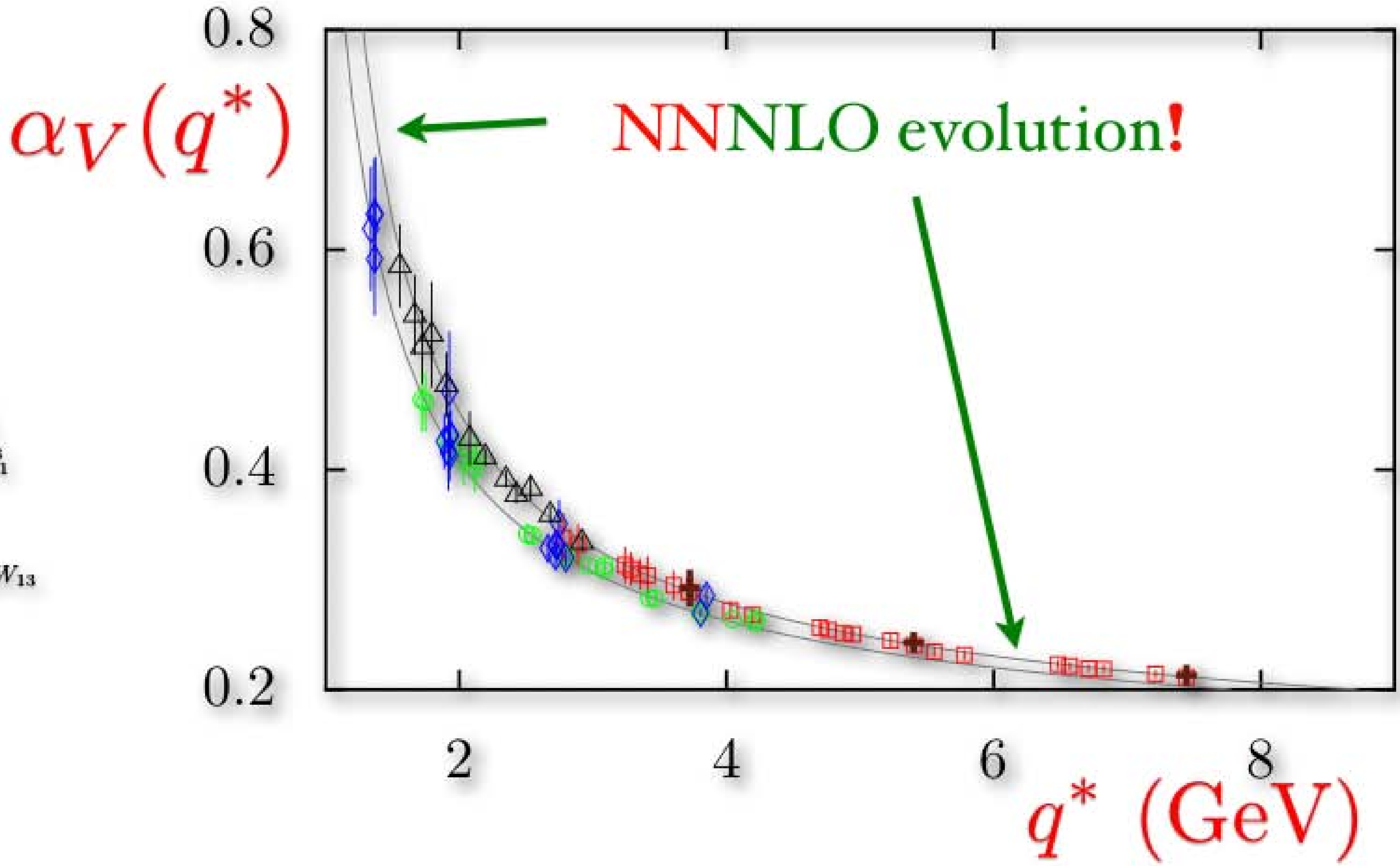
$$\langle \mathcal{O} \rangle = c_1 \alpha(q^*) + c_2 \alpha^2(q^*) + c_3 \alpha^3(q^*) + \ldots$$

(iv) Evolve $\alpha(q^*)$ to $\alpha_{\overline{\mathrm{MS}}}(M_Z)$

Some of the NNLO Diagrams



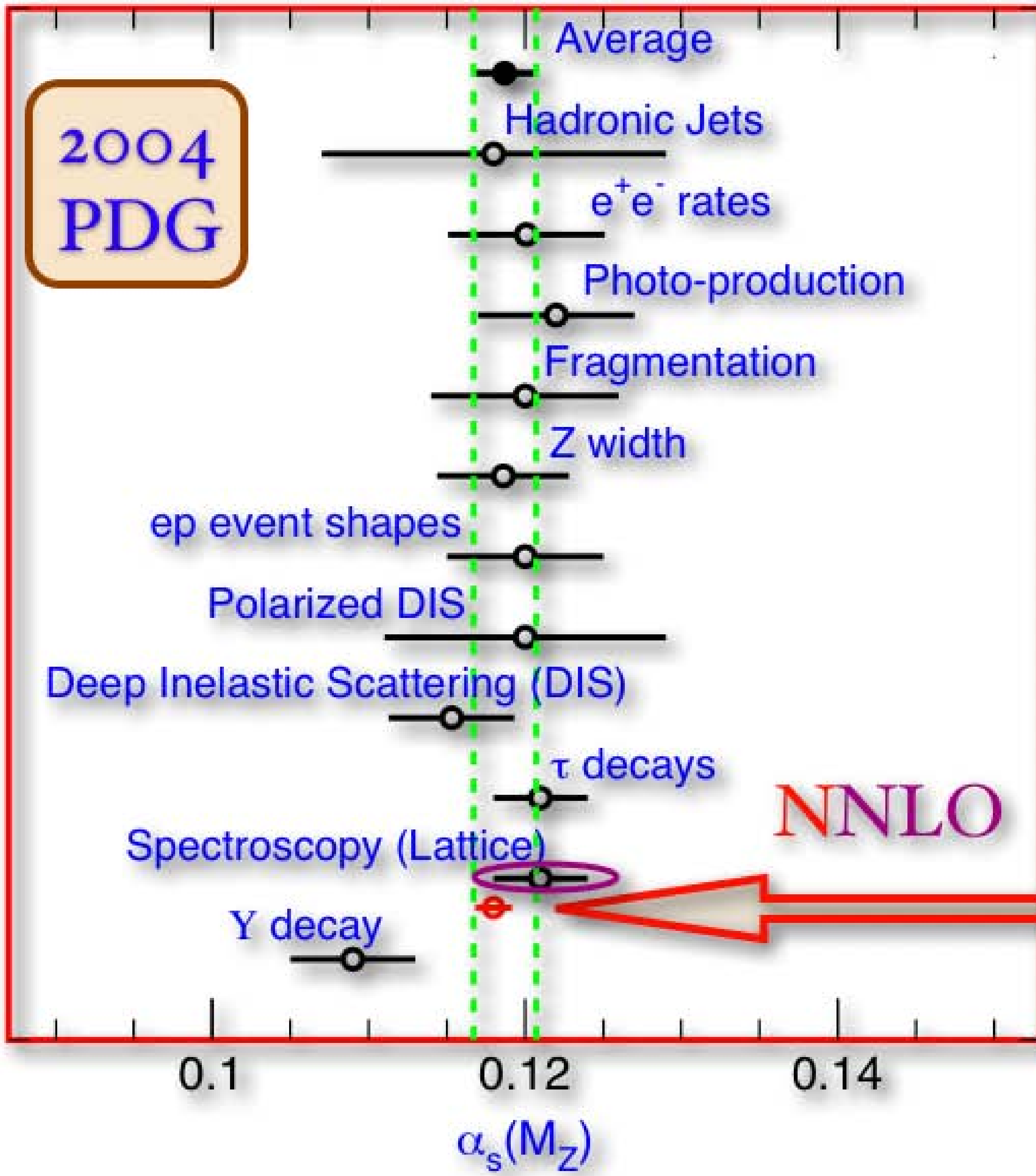




Error Budget

	$\log W_{11}$	$\log W_{13}/W_{22}$	$V(\sqrt{2}a) - V(a)$
a^{-1}	0.0007	0.0010	0.0010
$c_1 \ldots c_3$	0.0001	0.0004	0.0004
c_n for $n \geq 4$	0.0008	0.0005	0.0004
$V \to \overline{\text{MS}} \to M_Z$	0.0001	0.0001	0.0001
condensate	0.0002	0.0001	0.0001
m_u, m_d, m_s	0.0004	0.0001	0.0001
m_c, m_b	0.0002	0.0002	0.0002
simulation errors	0.0000	0.0000	0.0002
total uncertainty	0.0012	0.0012	0.0012

Result of new NNLO analysis



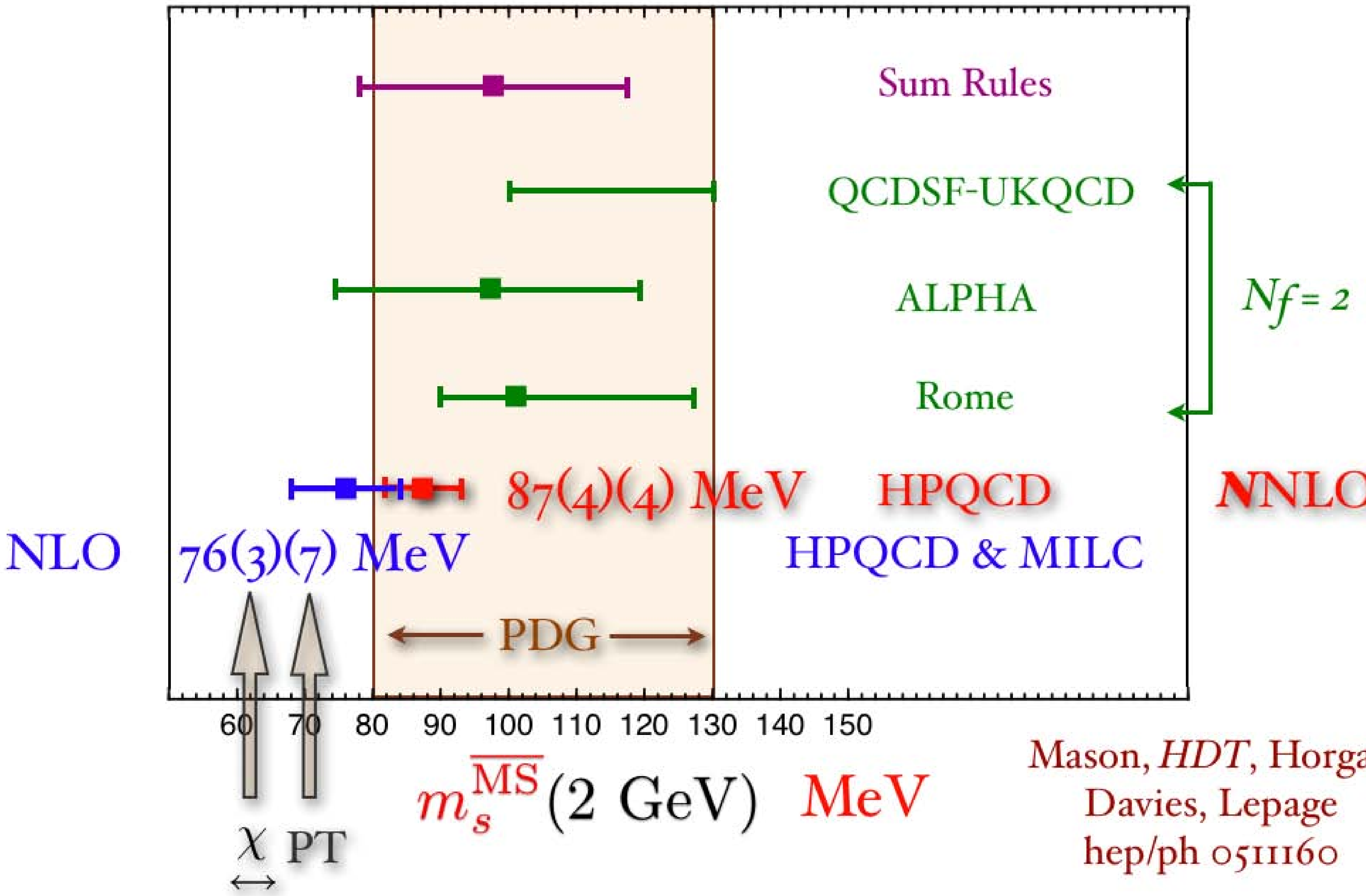
Mason, *HDT*, Davies, Foley, Gray, Lepage, Nobes, Shigemitsu PRL 95, 2005

Previous NLO result

$\alpha_{\overline{MS}}(M_Z)$ =0.1170(12)

PDG world avg = 0.1187(20)

(3b) NNLO light quark masses

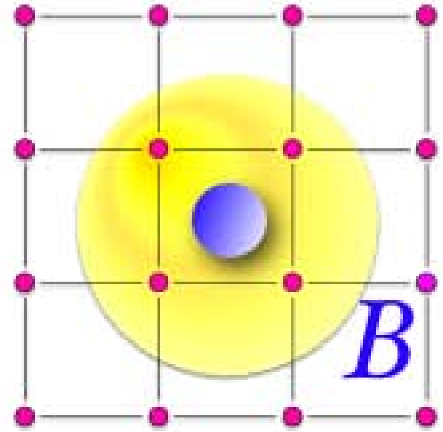


Mason, *HDT*, Horgan
Davies, Lepage
hep/ph 0511160

(3c) D & B meson decays

Heavy quarks on the lattice present another set of challenges

b quarks: $\frac{1}{m_b} \ll a$



Fortunately:

$\frac{\Lambda_{QCD}}{m_b} \ll 1$

Non-relativistic QCD (NRQCD)

Ex: (NLO)

$|V_{ub}| = 4.22(30)(51) \times 10^{-3}$

Gulez et al. hep-lat/0601021 (HPQCD: N_f=2+1)

c quarks:

Fermilab heavy-Q formalism

sums all orders in $m_Q\, a$

Ex: (NLO)

$f_D = 201(3)(17)\ \mathrm{MeV}$

Aubin et al., PRL 2005 (FNAL, MILC, HPQCD: N_f=2+1)

$= 223(17)(3)\ \mathrm{MeV}$ CLEOc

Relativistic approach to charm

- Use highly-improved staggered action (HISQ) developed for light quarks:

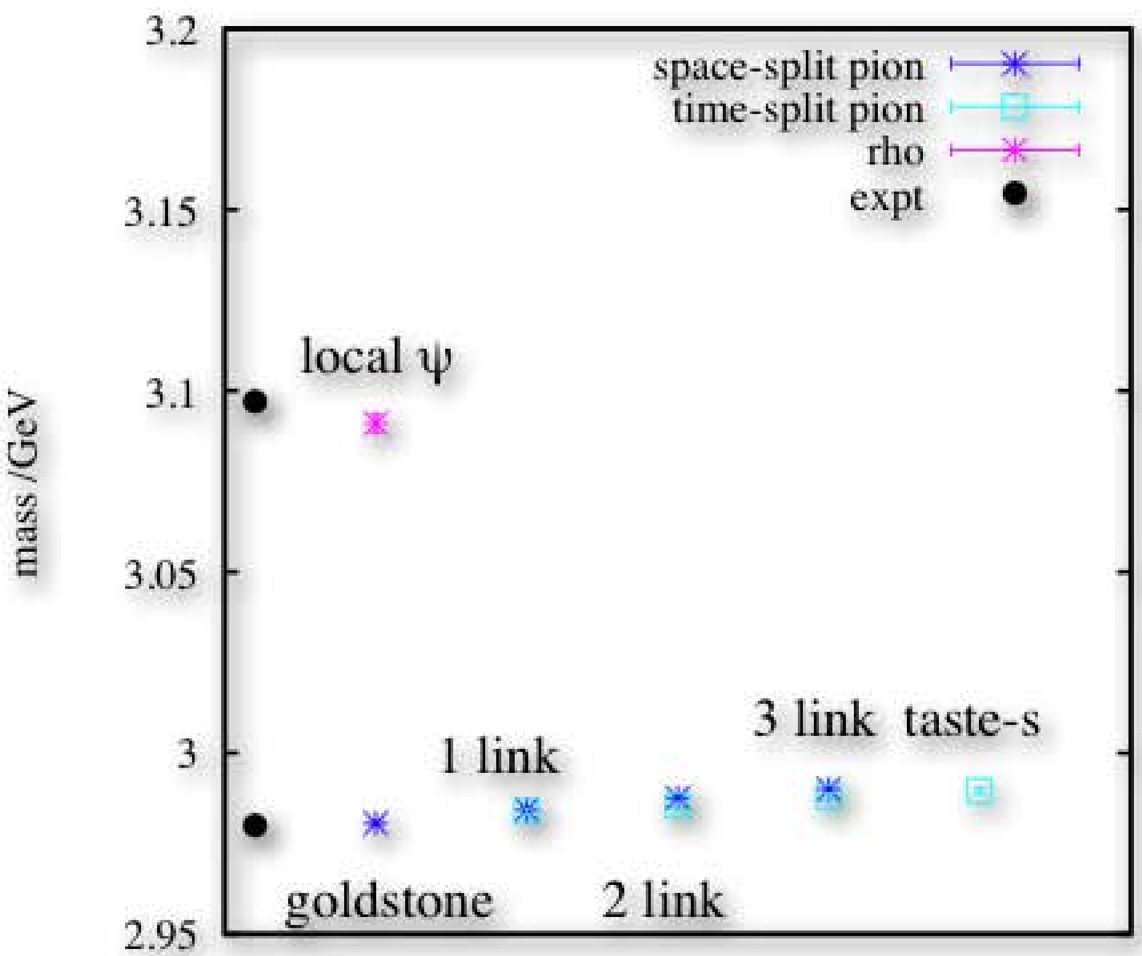
$O(a^4)$- and $O(\alpha_s a^2)$-accurate

- $m_c\, a$ ~ 0.4 on MILC fine lattices: ~5% accuracy!

Davies, Lepage, Follana, Wong

NNLO

f_D: *in progress!*



- taste-splittings (lattice artifacts) ~ 10 MeV

- $2m_{D_s} - m_{\eta_c} =$

preliminary

0.984(15)(43) GeV (HISQ)

0.956(1) GeV (Expt)

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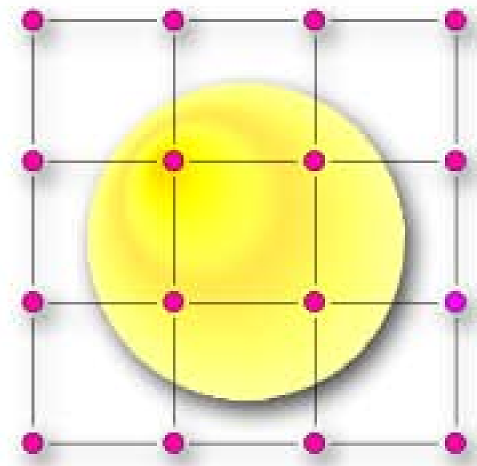
(4) "Chiral Symmetry vs. Lattice"

Creutz



fermion

Staggered Quarks

HPQCD, FNAL, MILC

Accept doublers: work hard to live with them

Wilson Fermions

CPU barrier

Still quenched & $O(a)$ →

Kill doublers, but break chiral symmetry

Luscher, Rome Group

- New simulation algorithm: *"Domain decomposition"*
  - UV & IR mode separation: dramatic performance gain
  - $m_\pi / m_\rho \sim 0.38$ ~ half previous Wilson quark mass **!**

Chiral Fermions

Ginsparg-Wilson (1982)

Two related schemes
under development for sometime:

When realistic QCD?

Start with vector-like theory, decouple mirror modes

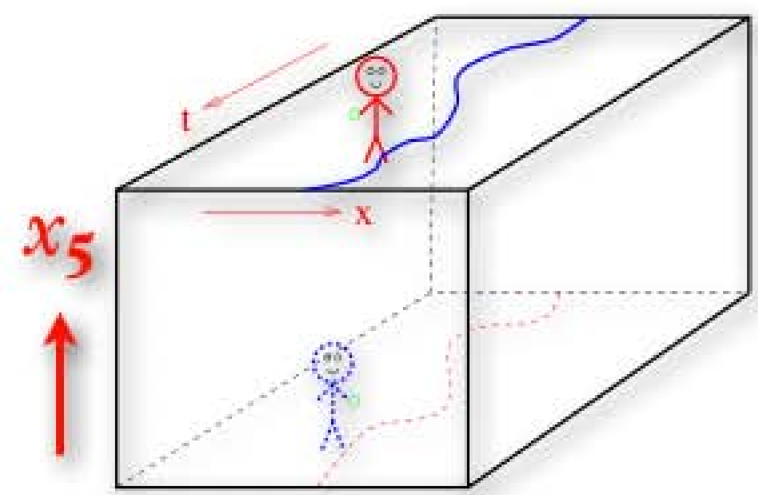
Domain wall: 5th dimension

(Kaplan 1992)

Still early days for

Dynamical Simulations: $N_f = 2+1$

RBC & UKQCD $m_u / m_s \sim 1/4$



Ripped from Creutz

Overlap fermions: integrate out mirror modes

(Narayanan & Neuberger 1993; Neuberger 1998)

highly nonlinear

Exploratory Dynamical Simulations: Wuppertal; Boulder

Summary

- Unquenched LQCD:
  few-% precision now demonstrated for many quantities
- HPQCD uses staggered quarks: residual issues
  - Perturbation theory key to most phenomenology
- Some important high precision applications done
  - NNLO $\alpha_{\overline{\rm MS}}(M_Z)$ ✔
  - NNLO m_s, m_c, m_b ✔
- Golden opportunity: precision measurements @ CLEO-c
  - In progress: *D*-decays, incl. $f_{\rm D}$ to ~ 5% this summer
- Other approaches to realistic LQCD: results *may* be coming?