Vertex finding prior to tracking in magnetic field

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The "Integral mathematics model of the track pattern" allows a computer to make a global overview of the whole hit array like a human does it before a reconstruction of the trajectories.

It gives the position of the primary intensive vertex, that allows to separate also the hits of multi vertices events (i.e. each vertex gets own set of hits).

The whole hit array can be subdivided on the "regions of interest" (in some limits of transversal momentum, for instance) and the reconstruction of trajectories in each region can be done at the more reliable conditions of smaller multiplicity in respect to the total multiplicity.

The recognition and analysis of the jets is also possible prior the tracking.

The ideas have been successfully tested by the D0 Monte-Carlo hits data.

All these possibilities simplify the following tracking and could be a base for the future developing of the high-level triggers.

1. **Mathematical model of the track pattern.**
2. **The Vertex functions /definition/.**
   a) **Straight tracks.**
   b) **Helix like tracks.**
3. **The “cleaning” of a vertex.**
4. **Accuracy of the z-vertex determination.**
5. **Location of jets (**preliminary**)**
6. **Kinematical separation.**
7. **What is left for the future.**
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1. Mathematical model of the track pattern

A most suitable word in the track recognition problem is, probably, a *DOUBT*. i. e. we know that there are some hits representing the trajectory but we are not very sure which one is associated to the trajectory. Thus we are saying:" this hit OR that hit OR..." belongs to trajectory.

In a mathematical formalism we can express these "OR's" as the sum of probabilities, that was done in the so called "Integral mathematical model of the track pattern"[1]

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$$R(c)=\sum_{n=1}^{N}\sum_{m=1}^{M_n}\exp(-(F(z_n;c)-a_{mn})^2/2\sigma^2) \qquad (1)$$

where: $F(z_n;c)$ is the function of ordinate z and set $\{c\}$ of parameters which describe the trajectory along of "z"; the $\{a_{mn};\ m=1,\ldots,M_n\}$ is the set of the M_n measured hits by a detector placed on $z_n(n=1,\ldots,N)$ where N is the total number of the detectors.

The DEFINITIONs are the very important step in the mathematical description of the track pattern. The introducing of the "small" indices "**mn**" at the measured X-coordinate "a_{mn}" reflects the multi hit character of pattern as well as the double summation "$\sum\sum$" sign reflects our double "OR" as well as the structure of the detecting system (from a formal point of view one can use one index, one sign of summation, but it would be the step back from the adequate description of the pattern, it can also produce additional combinatorial noise).

A straight line on the plane [XZ]:

$$x=F(z;A,B)=A\cdot z+B \qquad (2)$$

can demonstrate the character of this function "R(A,B)" (the magnetic field will be given a bit later):

$$R(A,B)=\sum_{n=1}^{N}\sum_{m=1}^{M_n}\exp(-(A\cdot z_n+B-a_{mn})^2/2\sigma^2) \qquad (3)$$

On the plane of the parameters **A,B** this function (3) looks like that:

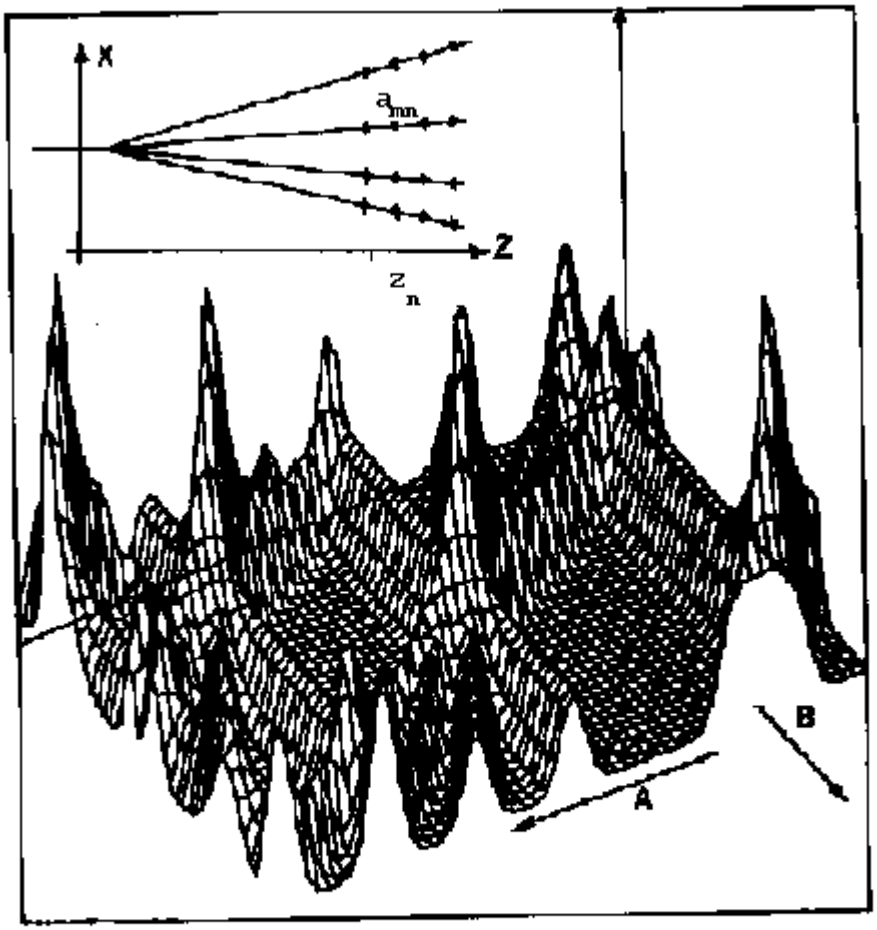

Fig 1. The view of R(A,B) (eq 3) for 16 hits Monte-Carlo event.

There are several big maximums and a lot of small ones, which reflect a noise or/and combinatorial background.
The amplitude of a "good" peak makes interesting junction of "**K**", that is number of hits along of a trajectory (there could be some "inefficient" detectors among of (**N**)) and the value of the Chi Square(χ^2):

$$R(c^*) \approx K - \chi^2(c^*)/2 \quad (K \leq N) \qquad (4)$$

Thus, if we have determined (by a mathematics) the position ($c^* \equiv (A^*,B^*)$ in our example) of the "good" peak we have got the "REAL" number of hits along the trajectory and its value of χ^2.

It is a good criterion in the recognition phase of the tracking for all the types **F(z;c)** of trajectories.

The χ^2 may not participate "alone" in the recognition , because it is defined on the base of logical AND (in opposition to a DOUBT it is a SURRANCE) whereas it some early to say that… does someone know what number(?) of hits should be taken, which hits ($a_{?n}$?).

$$\chi^2(c) = -\ln\{ \prod_{n=1}^{?} \exp(-(F(z_n;c) - a_{?n})^2/2\sigma^2) \} \qquad (5)$$

↑ it is logical "AND"

Thus we have the analytical function of the trajectory's parameters that includes itself all the detected hits; it joints the number of hits along a trajectory and its χ^2.

However, it is *difficult* (but *possible!*) to analyze this multi-extreme function.

2.The vertex functions.

A. The straight trajectories.

So a "by-product" had been extracted in as the "vertex functions"

Let we suppose (for straight tracks, again, - for a demonstration) that there is the primary vertex (X=x,Z=z) for some trajectories which left a "non-personalized" hit ($X=a_{kl}$, $Z=z_l$), so one writes:

$$A \cdot z + B = x \qquad (6)$$
$$A \cdot z_l + B = a_{kl}$$

That is the system for the re-definition of the “A,B” through unknown vertex “x,z”

$$A = (x - a_{kl})/(z - z_l)$$

$$B = (x \cdot z_l - a_{kl} \cdot z)/(z - z_l) \qquad (7)$$

Again we have a doubt which hit was produced by the vertex and which hit has other nature, thus we use again the logical “OR” ($\sum$) for all the hits ($k=1,\ldots,M_l$) of the detector (z_l). We can choose this detector (number ***l***), the first one if we are of interest of short lived particles; the last one (at Z_N) if we work for muons; all the detectors if they suffer of very low efficiency.

So the vertex function “D(x,z)” has the form:

$$D(x,z) = \sum_{k=1}^{M_l} \sum_{n=1}^{N} \sum_{m=1}^{M_n} \exp\left(-[(x-a_{kl})\cdot(z_n - z_l)/(z - z_l) + a_{kl} - a_{mn}]^2/2\sigma^2\right) \qquad (8)$$

and the view shown on Fig.2 (the maximum of this function equals really to the given position used on Fig.1).

If we were told that one position of vertex is known (x=0), then we can get simply the mathematical expression of the “**C(z)≡ D(x=0,z)**” /also: $x = A_\mu z + B_\mu$,for a muon physics/ (Fig.3).

Very important point: How to find the maximum of these functions?

If the variables of these functions are not separated (like D(x,z)) the only ***mathematical methods of the analysis of multi extrema functions*** are to be used.

If the variables can be separated or there is one variable only (like C(z)) then the ***histogram method*** can be used (the mathematics too): one calculates the value z*, which gives the maximum to the partial function of probability (“$\exp-[\ldots]^2/2\sigma^2$”) and puts this z* in the histogram(z).

HISTOGRAMM$[z^* = z_l + a_{kl}\cdot(z_n - z_l)/(a_{kl} - a_{mn})] \rightarrow C(z)$ **(9)**

This simple demonstration “how to build the vertex function for the straight tracks” is the general rule for other trajectories.

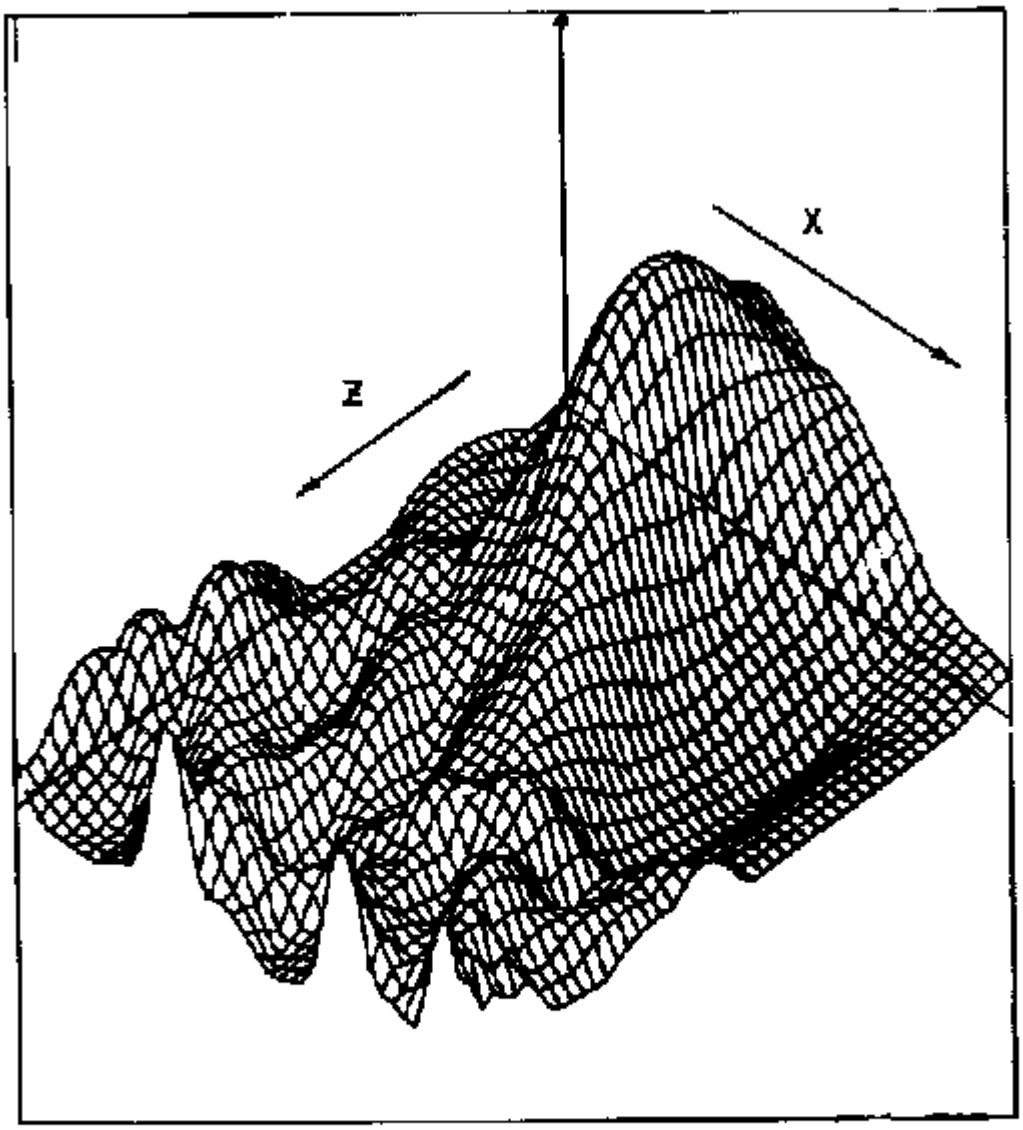

Fig 2. The view of D(x,z) (eq.8) for the hit pattern of shown on Fig.1

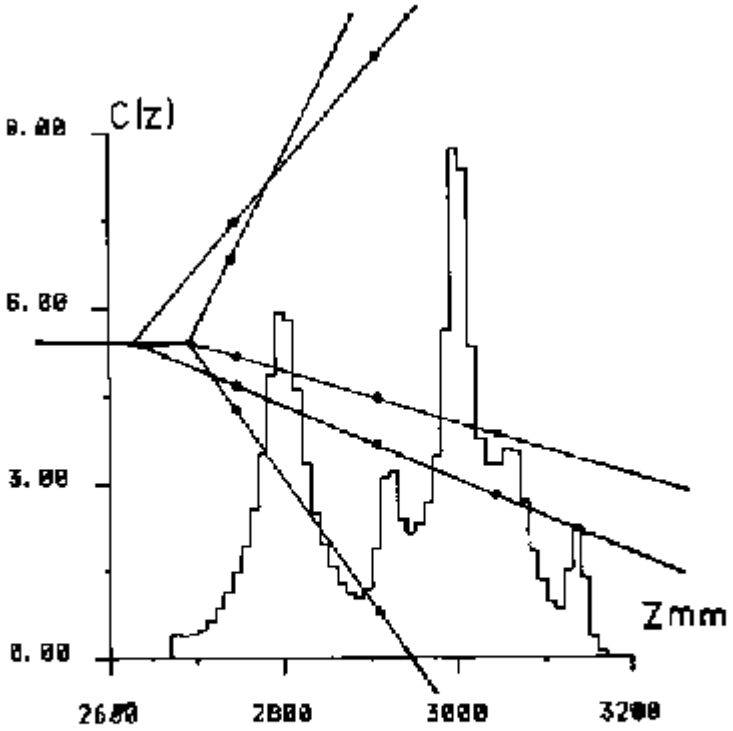

Fig 3.The view of C(z) for Monte-Carlo interactions of Mg: the first at z=2800mm; the second at z=3000mm.

B. The helix-like trajectories in magnetic field.

The definitions of this trajectory in XYZ /RφθZ system of coordinates:

$$x(z)=x_o+r_o\sin\alpha+r_o\sin[2A\cdot(z-z_o)-\alpha]\ (\equiv R\cos\varphi)$$

$$y(z)=y_o-r_o\cos\alpha+r_o\cos[2A\cdot(z-z_o)-\alpha]\ (\equiv R\sin\varphi) \qquad (10)$$

$$\alpha\equiv\varphi_o;\quad A\equiv eH/(2cP\cdot\cos\theta_o)\equiv eH/(2cP_L);\ r_o\equiv Pc\cdot\sin\theta_o/eH\equiv P_T\cdot c/eH$$

One can find in these formulas for the “straight” vertex function a traditional exhaustive search for the straight trajectories, however it becomes not so obviously for the magnetic field.

The vertex position (x_o,y_o,z_o) in the solenoid magnetic field is described [2] by the similar (to $D(x_o,y_o)$) function:

$$V(x_o,y_o,z_o)=\sum_{k=1}^{M_l}\sum_{n=1}^{N}\sum_{m=1}^{M_n}\exp(-[(F_{lk}-F_{mn})(z_o-z_k)/(z_k-z_n)+P_{lkmn}-F_{mn}]^2/2\sigma^2) \qquad (11)$$

where the parameters are defined as follows

$$P_{lkmn}\equiv Arctg[(y_{lk}-y_{mn})/(x_{lk}-x_{mn})]$$

$$F_{mn}(x_o,y_o)\equiv Arctg[(y_{mn}-y_o)/(x_{mn}-x_o)]. \qquad (12)$$

There (eq 11) is no any way to the track reconstruction, thus the $V(x_o,y_o,z_o)$ is an independent description of the track pattern.

The view of the $V(x_o,y_o,z_o=32.52$ cm) is shown on fig.4 There was the D0 Monte-Carlo event, the z_o-vertex was found, so this 2 dimensional plot(x_o,y_o) could be drawn.

The analysis of this function by the mathematics is the special story, - not now.

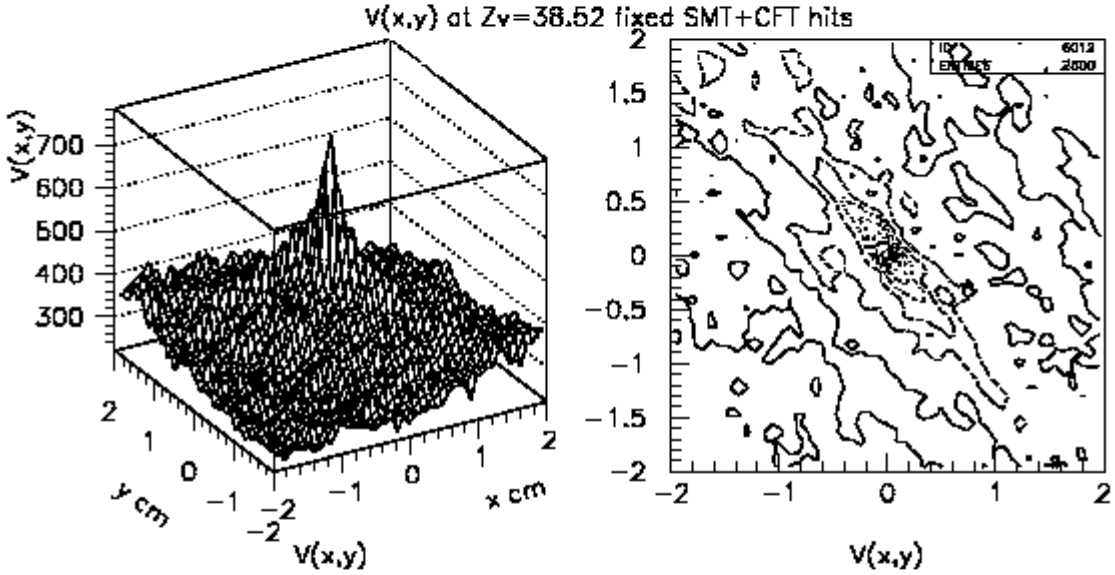

Fig.4 The view of the vertex function (eq11)on the plane (x,y) of possible positions of the vertex of a D0 Monte-Carlo event. The z-vertex was known (38.52 cm).

Fortunately, the beam physicists promise very good condition:

$$\mathbf{x_o=0} \quad \text{and} \quad \mathbf{y_o=0} \qquad (13)$$

So one can get the relatively simple $V(z_o)$ which can be analyzed both by the mathematics and the histogram method. The last one is simple and clear (but has his own problems, like a bin width related to the detectors accuracy), and gives some other possibilities.

3. The “cleaning” of a vertex.

When we have found the maximum of the histogram(z^*) which is the view of the vertex function $V(z_o)$ at σ=[bin width], we can identify the hits associated to this z-vertex. Thus it gives a chance to drop these hits in a separate array (to the tracking team) and remove these hits out from the original data set.

The rest of the hits can be analyzed again for other z-vertices until the last maximum is not less than a given threshold (we know the number of detectors and their efficiency).

Such a procedure of the “cleaning” of the vertices is demonstrated on fig.5: there were three Monte-Carlo vertices (two of them are a “minimum bias type”) generated for the D0 and all of them were successively found.

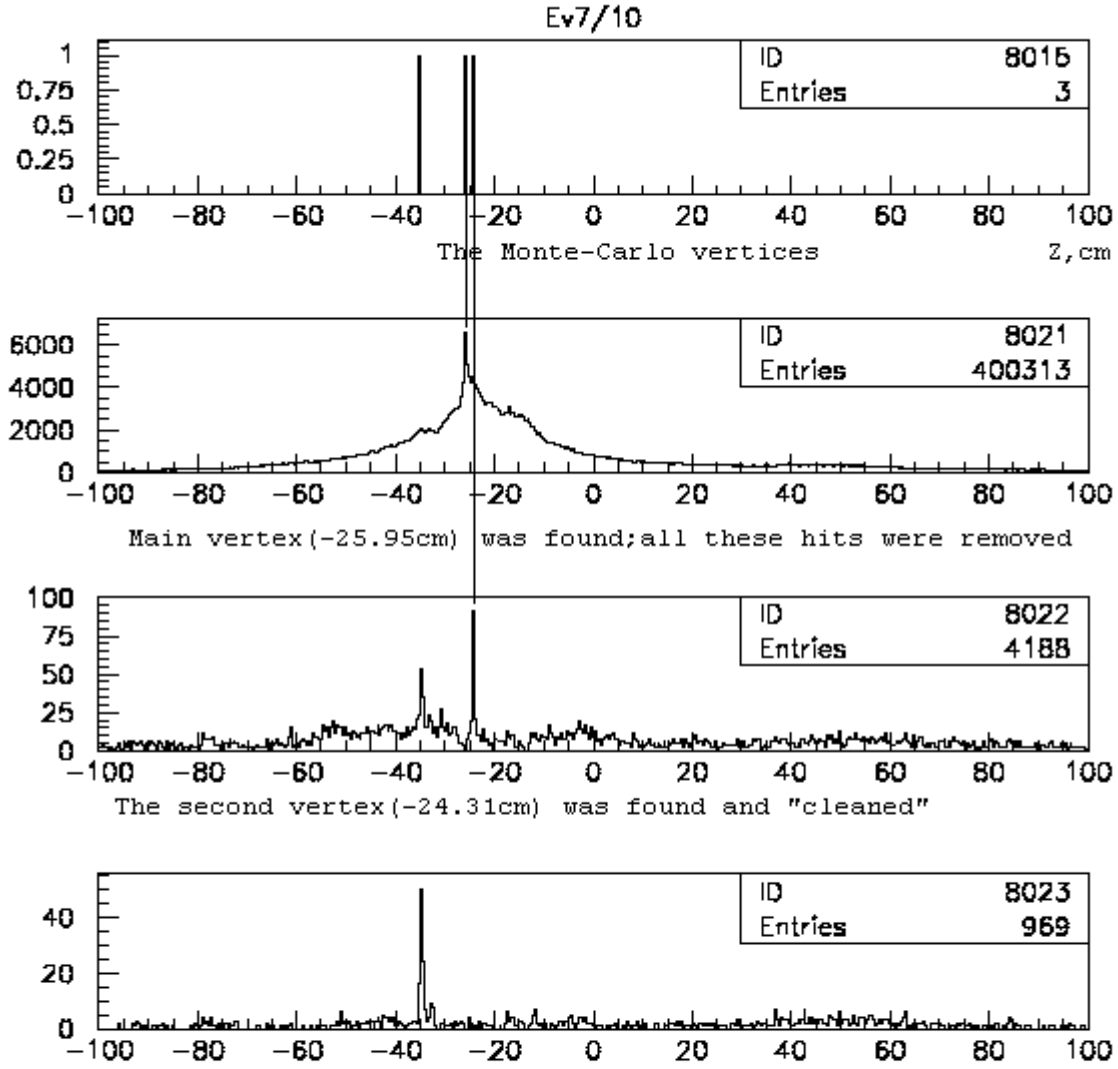

Fig.5 The vertex determination and cleaning for a D0 Monte-Carlo event.

4. Accuracy of the z-vertex determination.

There were analyzed 10 Monte-Carlo events generated for the D0 experiment. The average number of hits was about 1100 per event.

The deviation between the Monte-Carlo vertex and determined one by the histogram method (the z-maximum of the histogram) is –0.03+/-0.11 cm, it is not so bad for the vertex recognition for the following precise analysis by the traditional tracking.

However, there are some possibilities to increase the accuracy. After a first recognition of the z-vertex in a wide region of possible positions of the vertices, one can analyze this histogram narrowly. In particular the fitting by Gauss-like function can be applied. It gives more precise determination of the z-vertex,- for the same 10 events the average deviation has been improved: 0.007+/- 0.024 cm.

The example of such a zooming is shown on fig.6.

The accuracy is determined both by the detector accuracy, angular spreading of the particles, number of detectors(N) and multiplicity(M). The formal estimation of accuracy (σz_o) can be expressed in the terms of "longitudinal"(P_L) momentum and the angular accuracy (σ_φ), for instance:

$$\sigma z_o \approx (2c/eH) P_L \cdot \sigma_\varphi / \sqrt{MN} \qquad (14)$$

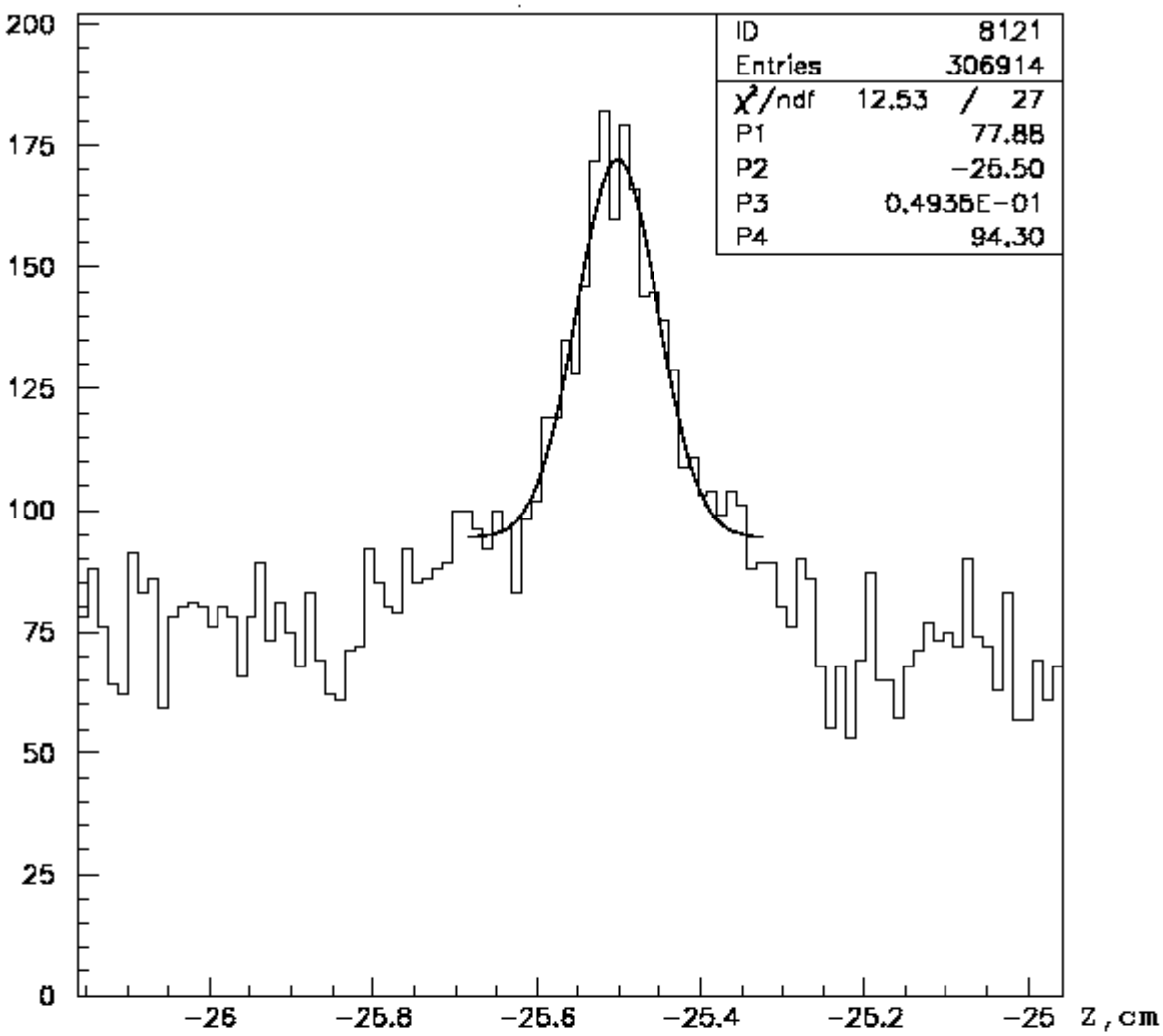

Fig.6 The fit (Gauss+Const) of the vertex function. It gives more precise detrmination of the z-vertex than it's done by the seraching of the z-maximum of histogram.

5. The determination of jets (preliminary)

When the z*-vertex value is determined the other kinematical parameters (eq.10) (α, θ_o, $\mathbf{r_o}$, $\mathbf{p_L}$, $\mathbf{p_T}$) are calculated by the way.

One can suppose that among of variety of those parameters there are some real ones.

Alike to the primary intensive vertex there can be defined other “large” object, - the jet which is emitted out from primary vertex.

These jets are characterized by the relatively narrow solid angle; the high-energetic part of a jet is also concentrated in a relatively small limits of $\mathbf{p_L}$, $\mathbf{p_T}$.

The analytical definition of these jets as a function of the kinematical parameters can be derived out from (1) similarly to the vertex definition (11). But fortunately these parameters are separated so one can apply the histogram method for the analysis of jets.

When the position of the primary vertex has been determined the kinematical parameters are re-calculated and booked in the one-dimensional or two-dimensional histogram.

One can suspect that the jet demonstrates itself as a high peak of the (α, θ_o)-histogram, for instance. The D0 Monte-Carlo events give this possibility a chance to be real one: there are two prominent peaks on the fig.7 in the histograms of α, θ_o, (α, θ_o).

This is a preliminary result because the comparison to the real jets has not been done yet.

However it gives a way to perform a macro analysis of the jets: if the direction (α, θ_o) of a jet has found, than one can use a calorimeter (an estimation of the energy deposition); the determined peak in solid angle gives also the estimation of the vector of the momentum ($\mathbf{P}$). All these values can used in the following precise tracking or can be applied to a kind of the jet trigger.

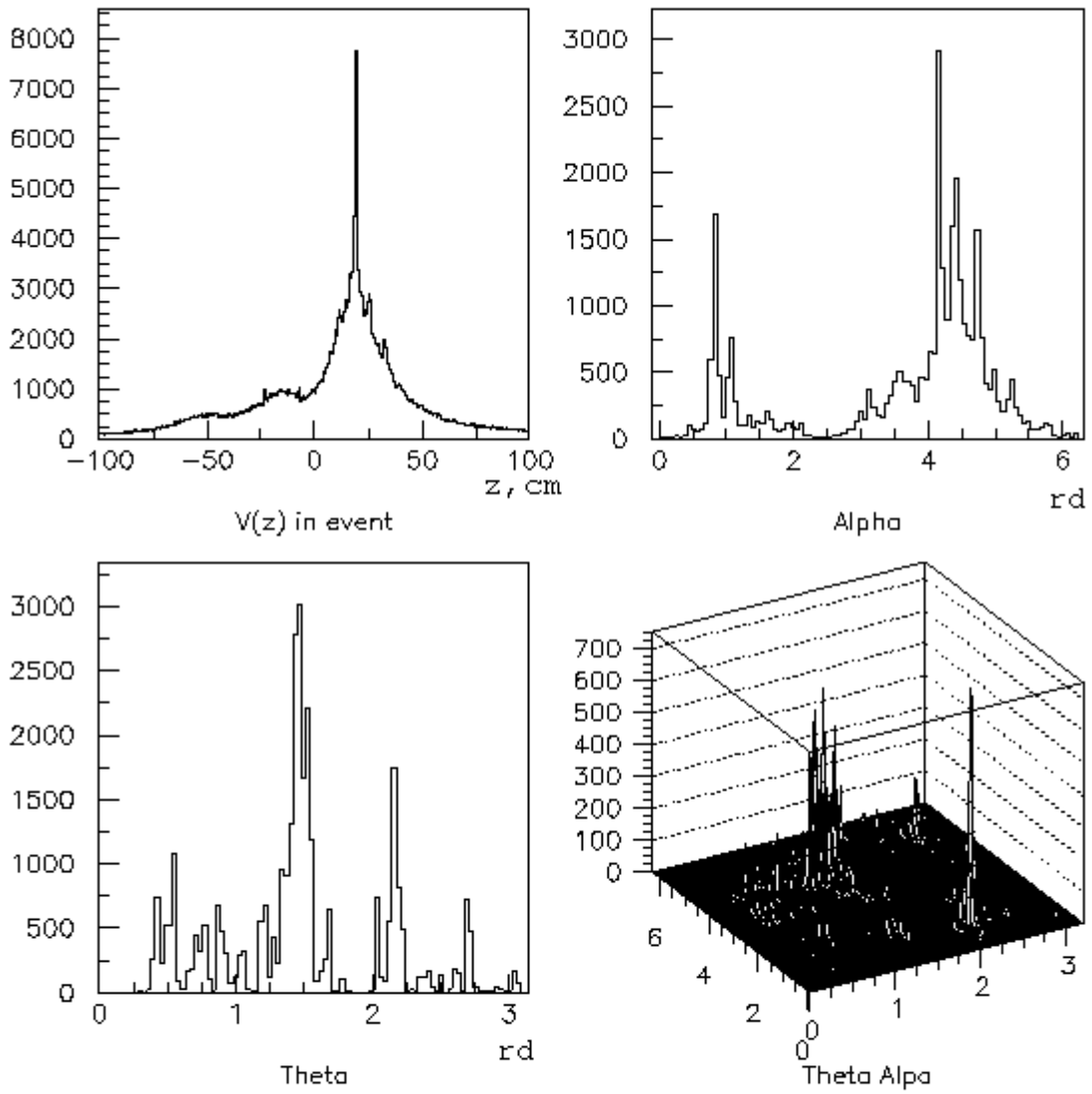

Fig.7 The upper plot shows the z-vertex histogram. For the hits assotiated to primary vertex the trajectory parameters (Alpha,Theta)are booked. The prominet peaks can evidence for the existing of jets.

6. The kinematical separation.

The “by-the-way” calculation of the kinematical parameters can simplify the life of the trackers.

When the parameters (α, θ_o), for example, have been found then the associated hits can be booked in (X,Y;R,Z)-histograms. As we are not much sure about a reality of these exact values, we determine the “*sub-region*” ($\alpha \pm \delta, \theta_o \pm \Delta$) where the relevant hits(x,y,z) will be booked in. There are shown on fig.8 the initial plot(x,y) for all the hits and three such the sub-regions. One can see that there is a more clear hit pattern in these sub-regions.

This can increase the reconstruction reliability and reduce the CPU-time. If there are “K” sub-regions for total number of hits “M”, then each sub-region has approximately M/K hits and a typical number of operations for tracking

$$T_s \approx (M/K)^2 = M^2/K^2$$

Then the total number of operations would be less in “K^2” times at a parallel computation and in **K** times at sequential tracking.

$$T_o \approx K * (M/K)^2 = M^2/K$$

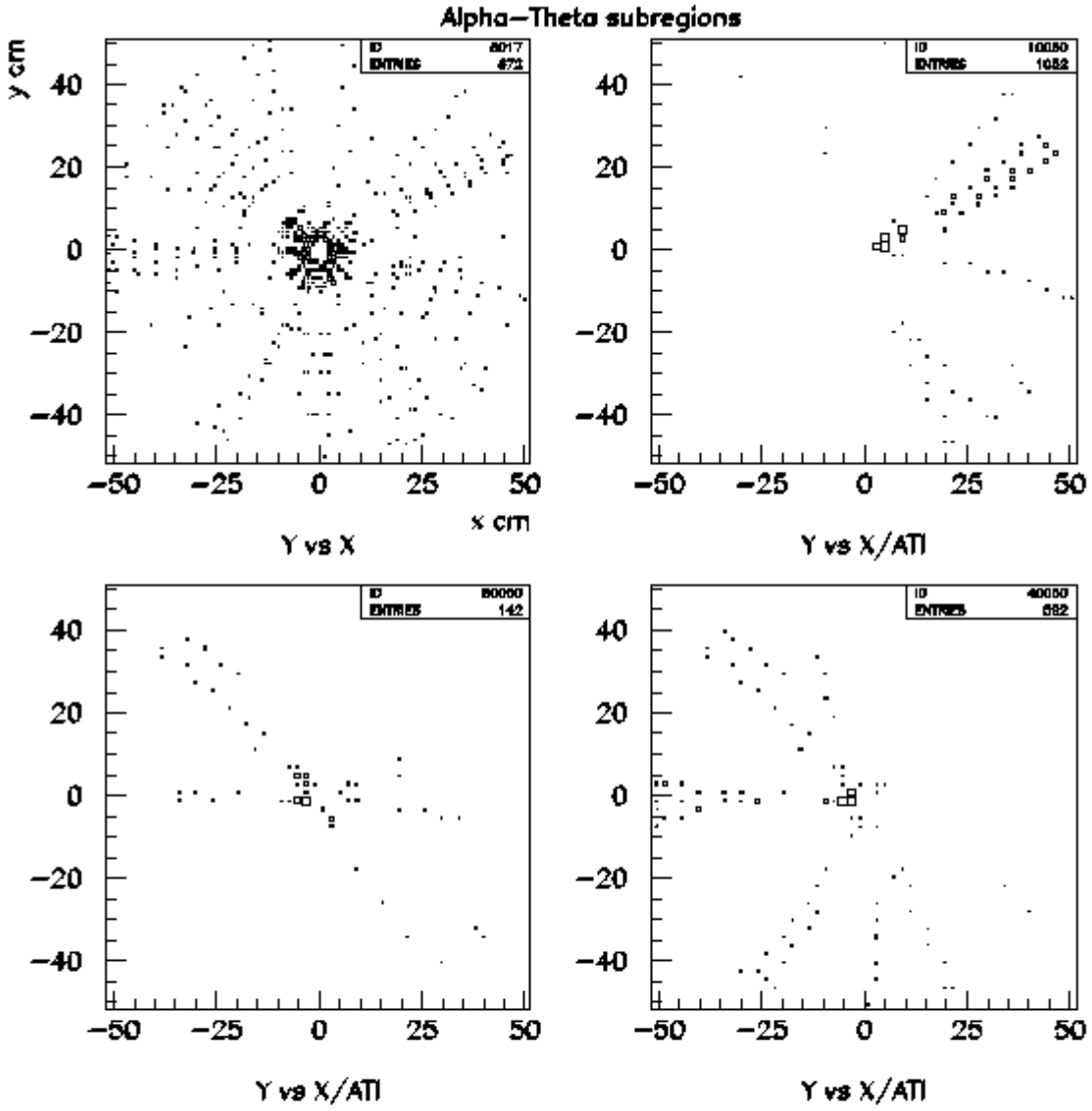

Fig.8 The upper left plots shows all the hits in (X,Y) plane; the other related to the hits associated to those Alpha,Theta parameters which are within the wide(60 degree) subregion of Alpha and15 degree of Theta.

7.CONCLUSION

A tracking, itself, requires sometimes the several re-analysis of whole track pattern to avoid the possible fake trajectories. To reduce this problem the next plan can be applied:

1) The determination of “invariants” of the pattern like the position of the primary intensive vertices.

2) To find the rest of the hits which are not associated to the primary vertex (the procedure of the “cleaning”) for the following searching for a rare decay, for instance.

3) To divide of the whole hit’s array into several “sub-regions” for the tracking at the comfortable conditions.

For the ***trigger*** purposes:

1) The “global “ analysis of the jets.

2) One can define a region of interest (on $\mathbf{P_T}$, for example) where there the integral over this region can be examined on maximum value.

3) This mathematics (1) allows one to describe formally a chain of decays, which is searching for (in the “rest” of the hits) prior the exact tracking.

Thus we have “algebra” for the track pattern.

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The References:

1.Yu.Yatsunenko,"Nuclear Instruments and Methods" A287 (1990) 422-430.

2. Yu.Yatsunenko, S. Selunin. The JINR Communication E1-92-333, JINR(Dubna, RU),1992.