Higher Order Calculations in the MSSM with FeynArts

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Fermilab, 10/00

1. Motivation
2. Techniques for two-loop calculations
3. Results for $\Delta\rho$, M_W, $\sin^2\theta_{\text{eff}}$
4. Conclusions

1. Motivation: Why MSSM?

The SM is in a pretty good shape.

Why Minimal Supersymmetric Standard Model (MSSM) ?

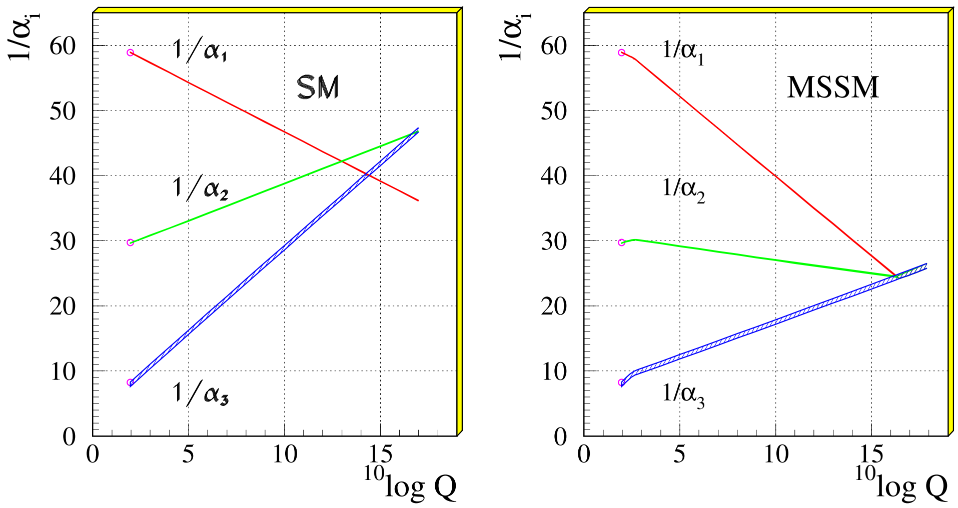
→ just 3 reasons (many more exist :-) :

1.) (Original motivation:) Stability of Higgs mass against higher order corrections in the MSSM

2.) Haag-Lopuszanski-Sohnius theorem:
maximal gauge symmetry for a QFT:
inner gauge symmetry $\otimes$ (local) Susy

3.) Unification of gauge couplings:
Not possible in the SM, but in the MSSM
(although it was not designed for it.)

Unification of the Coupling Constants
in the SM and the minimal MSSM



How can one distinguish SM and MSSM? Two possible ways:

- Search for new SUSY particles

new SUSY particles found $\Updownarrow$ SM ruled out

Problem:
SUSY particles are too heavy for todays colliders, only upper limits of $\mathcal{O}$(100 GeV).

Exception: mass of the lightest Higgs m_h

$\rightarrow$ two-loop calculations necessary
(see G. Weiglein's talk)
$\rightarrow$ MSSM with complex parameters

[*S.H. et al., work in progress*]

- Search for indirect effects of SUSY:
Precision Observables

Precision Observables (PO):

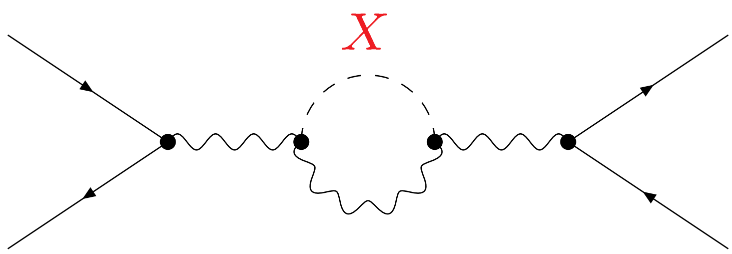
Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$\Delta\rho, M_W, \Delta r, \sin^2\theta_{\text{eff}}$	$\leftrightarrow$	SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



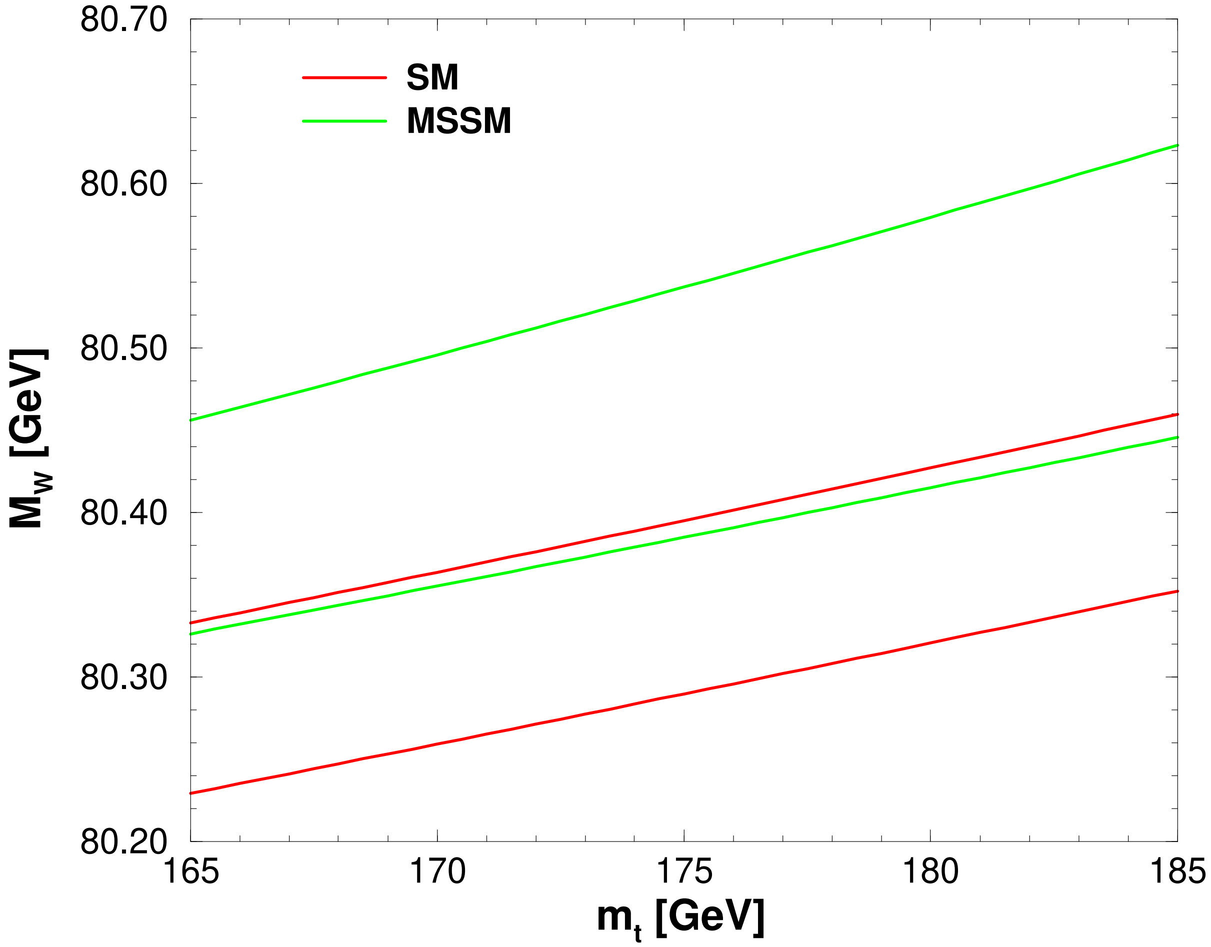
$\Downarrow$

Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

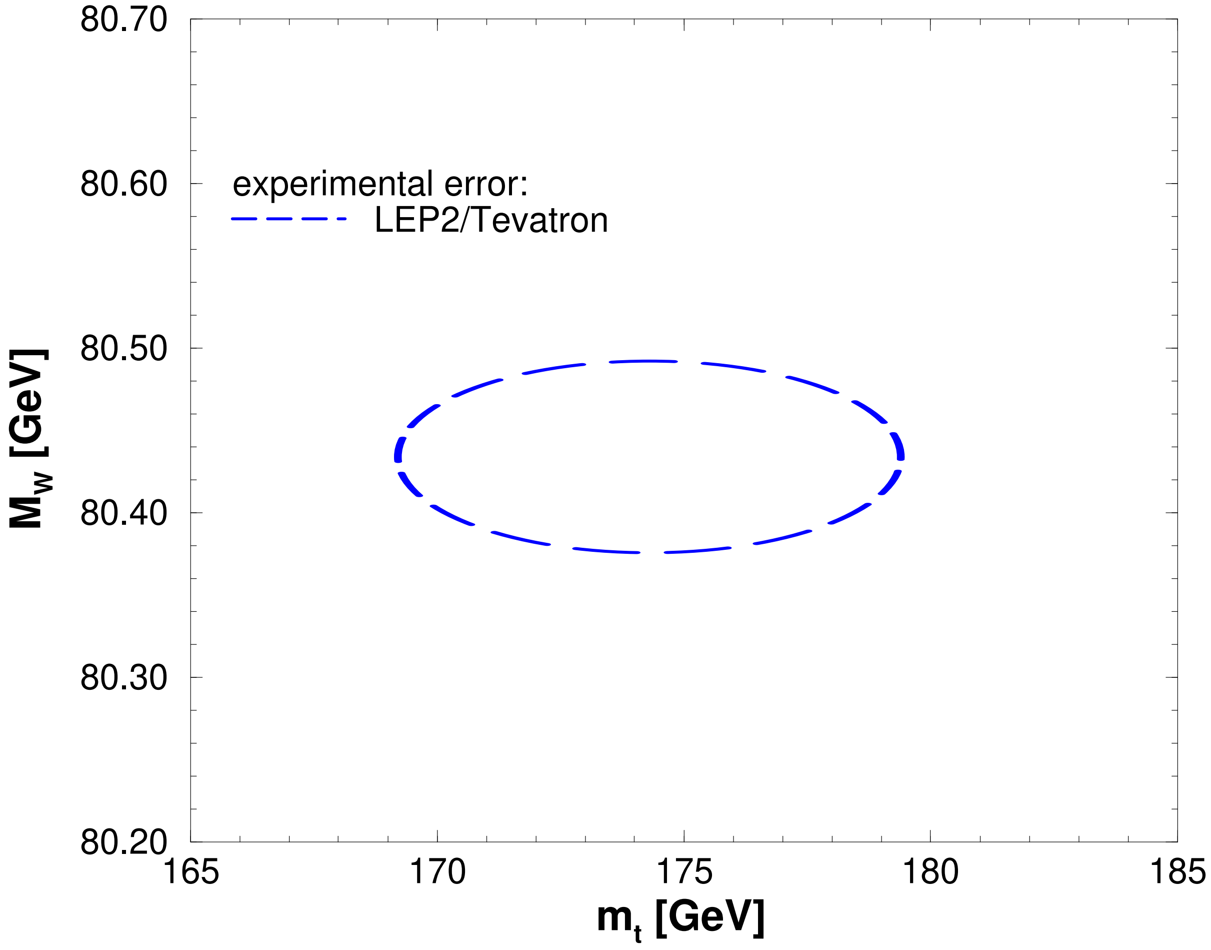
Example: W boson mass M_W $\rightarrow$ FF

Prediction for M_W in the SM and the MSSM :
Assumption: no SUSY particles found at LEP2

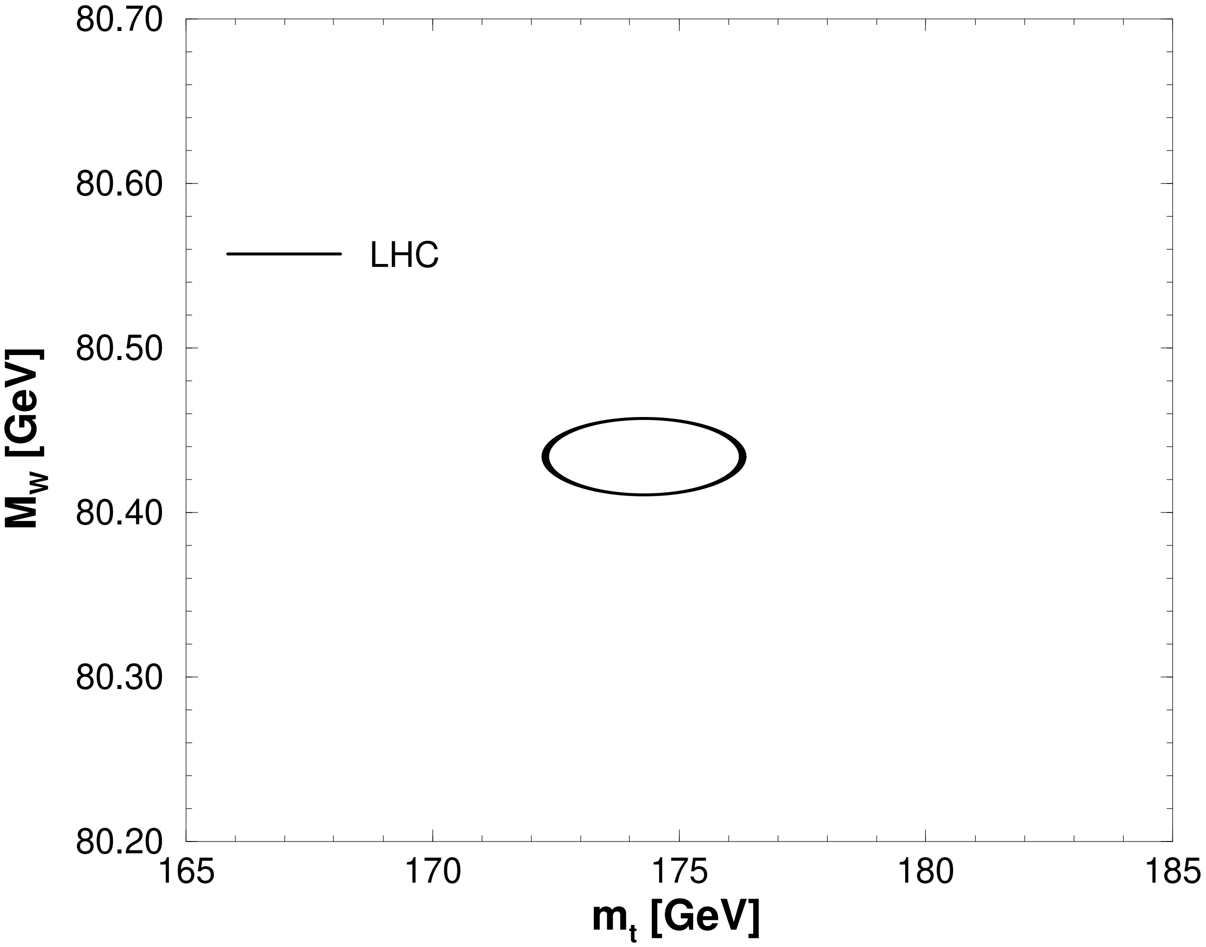


SM uncertainty: unknown Higgs mass

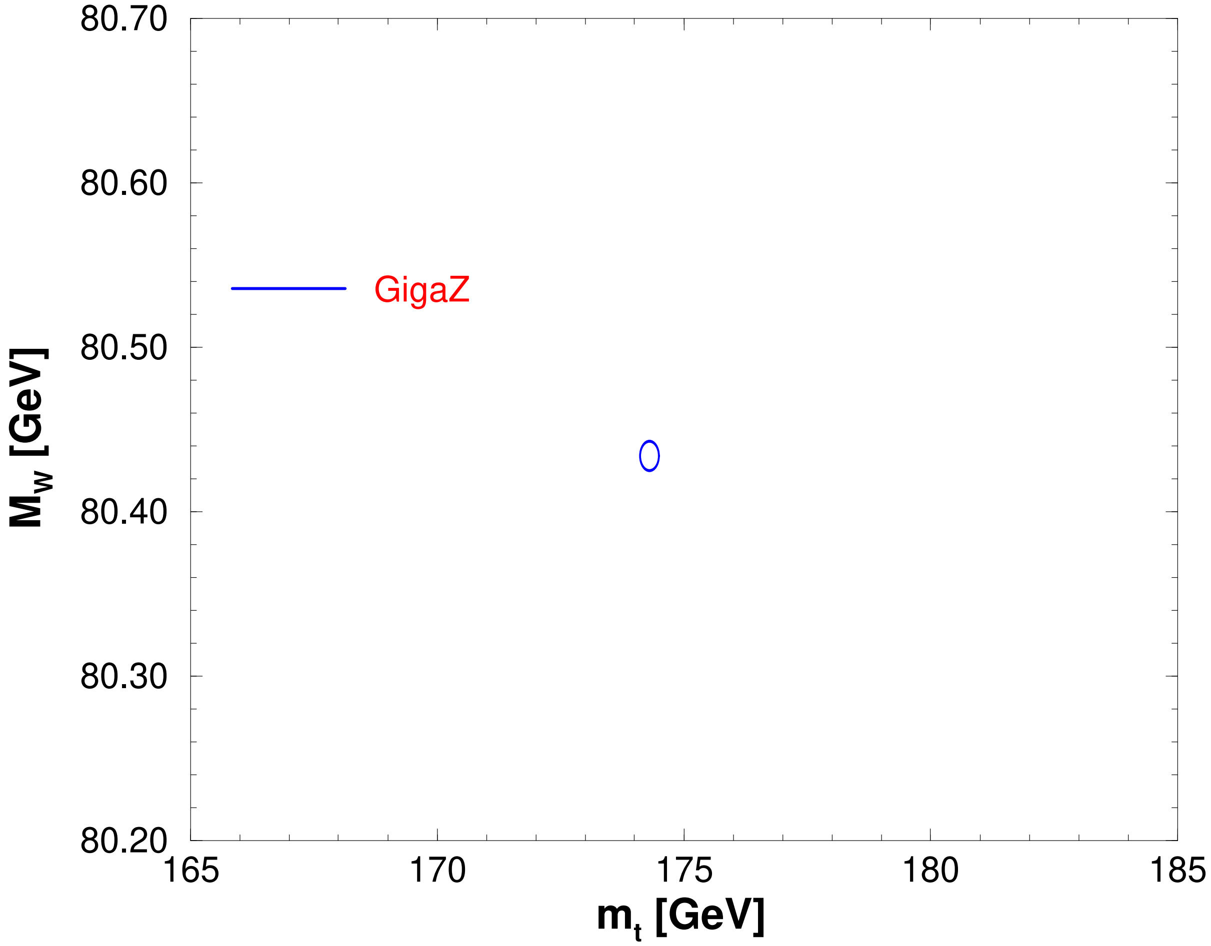
MSSM uncertainty: unknown masses of SUSY particles



80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
165
170
175
180
185
m_t [GeV]
experimental error:
LEP2/Tevatron



80.70
80.60
80.50
80.40
80.30
80.20
165
170
175
180
185
LHC
M_W [GeV]
m_t [GeV]


80.70
80.60
80.50
80.40
80.30
80.20
GigaZ
M_W [GeV]
165
170
175
180
185
m_t [GeV]

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$\left[u, d, c, s, t, b\right]_{L,R}$ $\left[e, \mu, \tau\right]_{L,R}$ $\left[\nu_{e,\mu,\tau}\right]_L$ Spin $\frac{1}{2}$

$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R}$ $\left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R}$ $\left[\tilde{\nu}_{e,\mu,\tau}\right]_L$ Spin 0

g $\underbrace{W^\pm, H^\pm}$ $\underbrace{\gamma, Z, H_1^0, H_2^0}$ Spin 1 / Spin 0

$\tilde{g}$ $\tilde{\chi}^\pm_{1,2}$ $\tilde{\chi}^0_{1,2,3,4}$ Spin $\frac{1}{2}$

Enlarged Higgs sector (two Higgs doublets):

physical states: $h^0, H^0, A^0, H^\pm$
Goldstone bosons: $G^0, G^\pm$

Additional input parameters: $\tan\beta$, M_A

Problem in MSSM: <u>many scales</u>

The ρ-parameter:

ρ measures the relative strength between
neutral current interaction and
charged current interaction .

$$\rho = \frac{1}{1 - \Delta\rho} \qquad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

$$\Delta M_W \approx \frac{M_W}{2} \frac{c_w^2}{c_w^2 - s_w^2} \Delta\rho,$$

$$\Delta \sin^2 \theta_W^{\text{eff}} \approx -\frac{c_w^2 s_w^2}{c_w^2 - s_w^2} \Delta\rho$$

Why $\mathcal{O}(G_F^2 m_t^4)$ corrections?

- First SUSY contributions of $\mathcal{O}(m_t^4)$

  Same size as $\mathcal{O}(\alpha\alpha_s)$ corrections?

- Up to now:
  SM result used with $M_H^{\text{SM}} \to m_h^{\text{SUSY}}$

  Correct?

2. Techniques for two-loop calculations

Heavy use of computer algebra programs:

- Automatic generation of Feynman diagrams and amplitudes: *FeynArts*
  [*J. Küblbeck, A. Denner, M. Böhm '90*]
  [*H. Eck '95*]
  [*T. Hahn '98, '00*]

- Algebraic evaluation and tensor integral reduction to scalar integrals: *TwoCalc*
  (works for two-loop self-energies)
  [*G. Weiglein '92*]
  [*G. Weiglein, R. Scharf, M. Böhm '94*]

- Further evaluation: insertion of integrals, expansion in $\delta = \frac{1}{2}(4 - D)$
  $\to$ algebraical check: cancellation of divergencies

- Result:
  - algebraic *Mathematica* code
  - Fortran code

- Numerical evaluation:
  $\to$ numbers, plots etc

Status of FeynArts:

*FeynArts*2.2: [*T. Hahn '98, '00*]

- model file:
  all information about particles and couplings
  MSSM model file: finished
  [*T. Hahn, C. Schappacher '00*]

  (some) CTs are missing

  $\to$ ready for one-loop calculations
  $\to$ limited application for two-loop calculations possible

- checks performed:

$$
\begin{aligned}
e^+e^- &\to t\bar{t} \\
e^+e^- &\to H^+H^- \\
q\bar{q} &\to t\bar{t} \\
e^+e^- &\to \tilde{\chi}^{0,+}\tilde{\chi}^{0,-}
\end{aligned}
$$

  `www.hep-processes.de`
  [*T. Hahn et al. '99, '00*]

*FeynArts*3β: [*T. Hahn '00*]

- new graphics features

Everything available at: `www.feynarts.de`

Generation of Feynman diagrams with *FeynArts*:

Example: Z boson self-energy:
(Mathematica session)

```
> <<FeynArts.m;

> se2 = CreateTopologies [2, 1->1,
        ExcludeTopologies->Internal];

> V2V2 = InsertFields [se2, V[2] -> V[2],
         Model->"MSSM", InsertionLevel->Particles,
         ExcludeParticles -> { ... } ];

> Paint[V2V2];                                  → F

> V2V2A = CreateFeynAmp [V2V2];

> V2V2A >> V2V2.amp;
```

`V2V2.amp` contains Z boson self-energy amplitude at the two-loop level

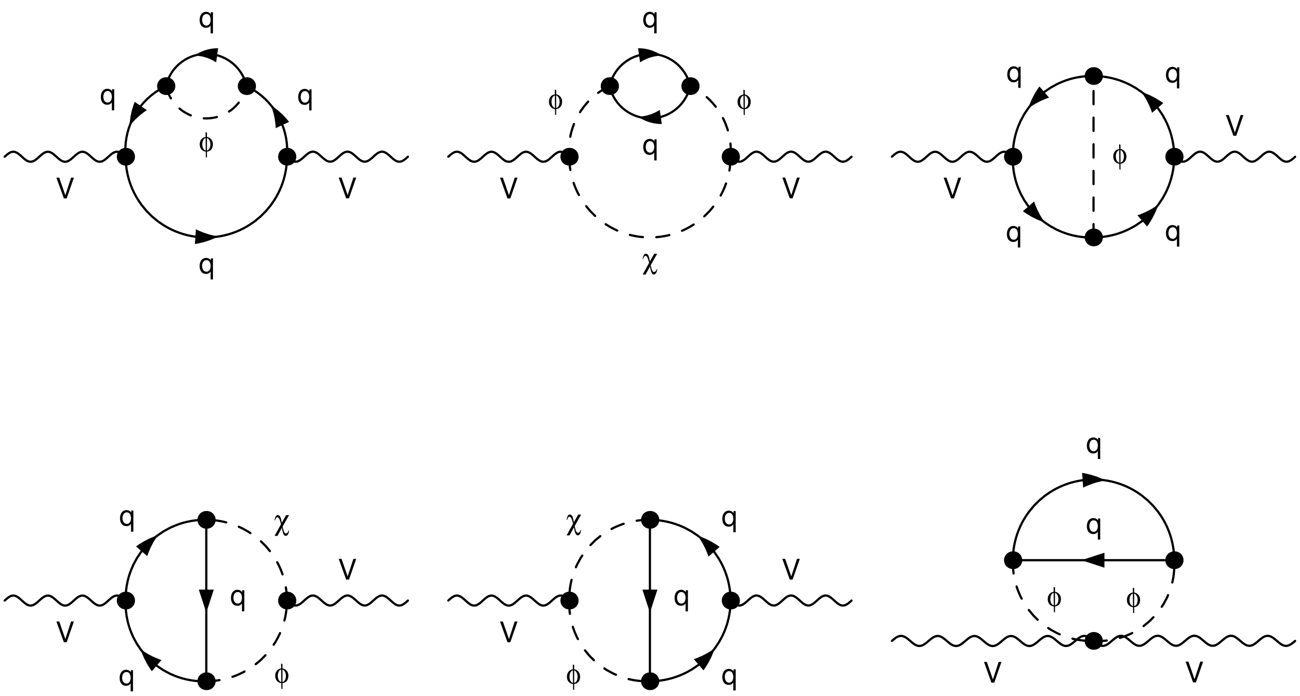
Same for

– other two-loop diagrams

– one-loop diagrams with CT insertion → F

– one-loop CT diagrams → F

Two-loop diagrams:

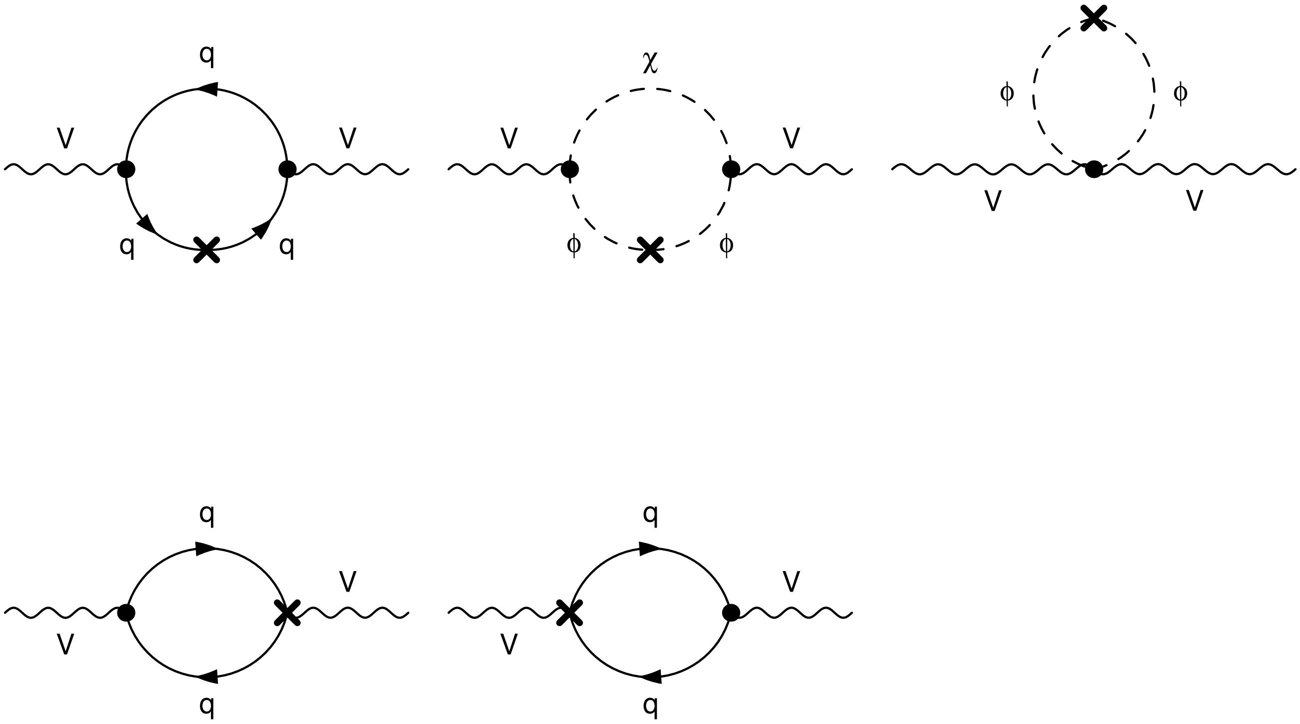


$$V = Z, W^{\pm}$$

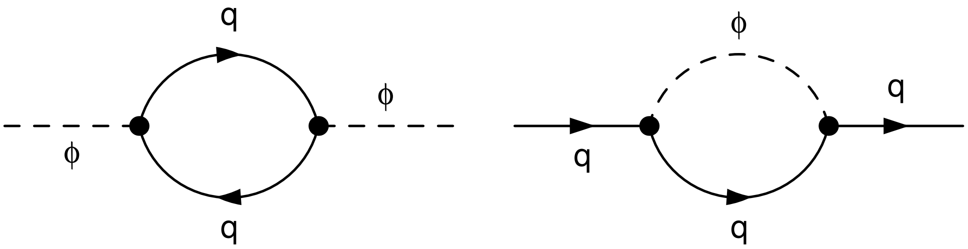
$$q = t, b$$

$$\phi, \chi = h^0, H^0, A^0, H^{\pm}, G^0, G^{\pm}$$

Counterterm insertions:



CT diagrams:



Further evaluation with *TwoCalc*:

Example: Z boson self-energy:
(Mathematica session)

```
> <<TwoCalc.m;

> amp =<<V2V2.amp;

> SetOptions[ TwoLoop, Dimension -> $D,
              CollectFunction -> 0];
```

(CollectFunction: DRED implemented)

```
> res = TwoLoopSum [amp, SelfEnergyPart -> 1];

> res = Collect[res,
              {A0[_], T[__]}, Simplify];

> res >> V2V2.res;
```

`V2V2.res` contains Z boson self-energy amplitude at the two-loop level in terms of A_0 and T functions
($T \mathrel{\widehat{=}} T_{134}$: two-loop integral)

Further evaluation:

– check for finiteness

– insertion of integrals

$\Rightarrow$ finite result as Mathematica code

3. Results for $\Delta\rho, M_W, \sin^2\theta_{\mathrm{eff}}$

Finite result for $\Delta\rho^{\mathrm{SUSY}}$ in terms of $s_\beta = \tan\beta/\sqrt{1+\tan^2\beta}$ and $a \equiv m_t^2/M_A^2$:

$$
\begin{aligned}
\Delta\rho_1^{\mathrm{SUSY}} = 3\, \frac{G_F^2}{128\,\pi^4}\, m_t^4\, \frac{1-s_\beta^2}{s_\beta^2\, a^2} \times \\
\Big\{ &\mathrm{Li}_2\left((1-\Delta)/2\right) \frac{8}{\Delta}\Lambda \\
&-2\,\mathrm{Li}_2\left(1-\frac{1}{a}\right)\left[5 - 14a + 6a^2\right] \\
&+\log^2(a)\left[1+\frac{2}{\Delta}\Lambda\right] - \log(a)\left[2 - 20\,a\right] \\
&-\log^2\left(\frac{1-\Delta}{2}\right)\frac{4}{\Delta}\Lambda \\
&+\log\left(\frac{1-\Delta}{1+\Delta}\right)\Delta(1-2\,a) \\
&-\log\left(|1/a - 1|\right)\,(a-1)^2 \\
&+\pi^2\left[\frac{2\Delta}{-3+12\,a}\Lambda + \frac{1}{3} - 2\,a^2\frac{s_\beta^2}{1-s_\beta^2}\right] \\
&-17a + 19\frac{a^2}{1-s_\beta^2}\Big\}
\end{aligned}
$$

with $\Lambda = 3 - 13a + 11a^2$ and $\Delta = \sqrt{1-4a}$

Expansion for large M_A:

$$\Delta\rho_1^{\mathrm{SUSY}} = 3\,\frac{G_F^2}{128\,\pi^4}\,m_t^4 \times$$

$$\Bigg\{ 19 - 2\pi^2$$

$$-\frac{1-s_\beta^2}{s_\beta^2}\Bigg[\left(\log^2 a + \frac{\pi^2}{3}\right)$$

$$\left(8a + 32a^2 + 132a^3 + 532a^4\right)$$

$$+\log(a)\frac{1}{30}\Bigg(560a + 2825a^2$$

$$+ 11394a^3 + 45072a^4\Bigg)$$

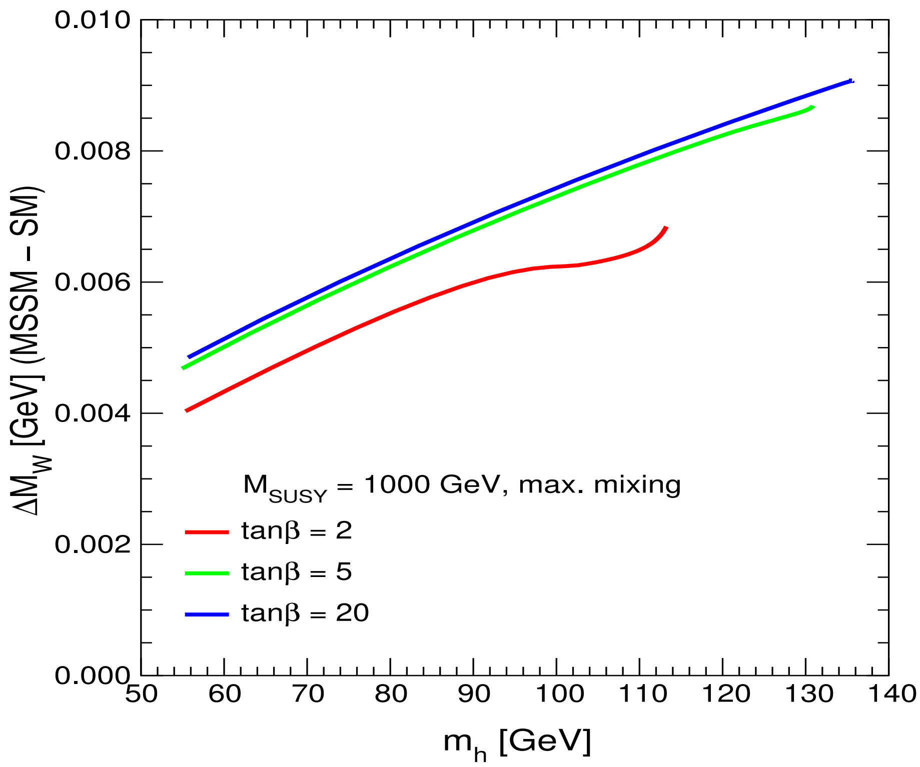
$$-\frac{1}{1800}\Big(2800a + 66025a^2 + 300438a^3$$

$$+ 1265984a^4\Big) + \mathcal{O}\left(a^5\right)\Bigg]\Bigg\}$$

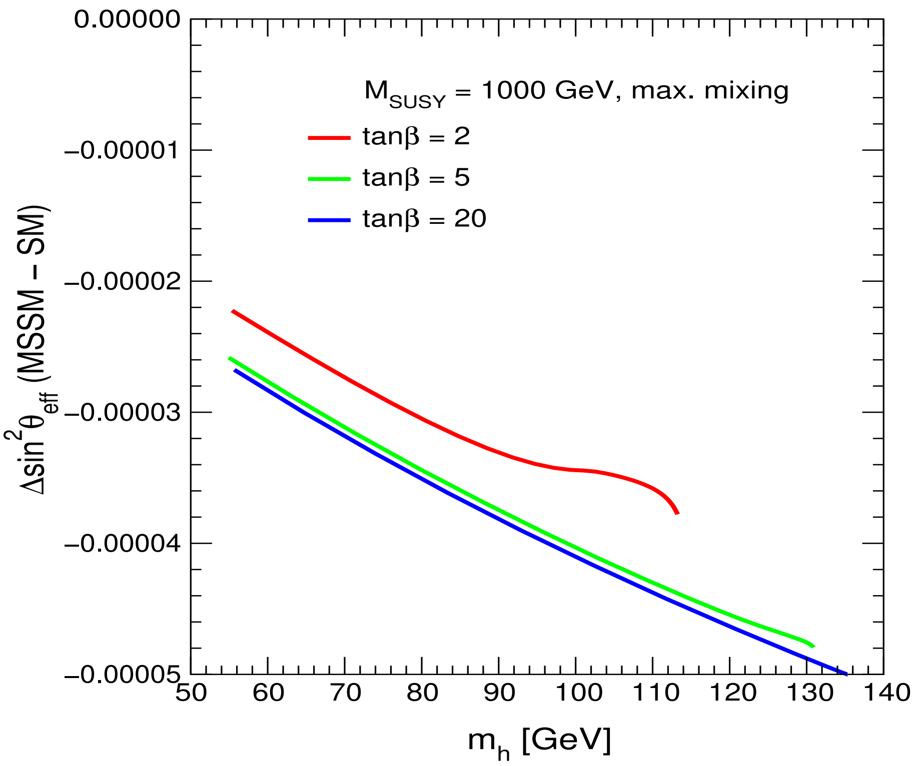
Check:

For $M_A \to \infty$ SM result obtained with $M_H^{\mathrm{SM}} \to 0$

Numerical result for $\Delta M_W^{\mathrm{SUSY}}$:



Numerical result for $\Delta \sin^2 \theta_{\mathrm{eff}}^{\mathrm{SUSY}}$:



4. Conclusinos

- Calculation of $\Delta\rho^{\text{SUSY}}$ in $\mathcal{O}(G_F^2\, m_t^4)$
  (limit of $M_{\text{SUSY}} \to \infty$)
- Heavy use of computer algebra programs:
  - *FeynArts* (Generation of MSSM diagrams)
    - $\to$ MSSM model file finished
      (ready for one/two-loop calculations)
    - $\to$ many checks performed
      (complete processes)
    - $\to$ *FeynArts3*β available
  - *TwoCalc* (Reduction to scalar integrals)
- Compact formula in terms of
  $\tan\beta$ and $a = m_t^2/M_A^2$
  (SM limit obtained for $M_A \to \infty$)
- Numerical results:
  larger result for effective change SM $\to$ MSSM
  - ΔM_W^{SUSY}: $\lesssim 10$ MeV
  - $\Delta \sin^2\theta_{\text{eff}}^{\text{SUSY}}$: $\lesssim -5 \times 10^{-5}$

Further information:
`www.feynhiggs.de, feynarts.de, hep-processes.de`