Gauge Invariant Classes of Feynman Diagrams and Applications for Calculations

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Outline

- Motivation
- Simple examples
- "Forests and Groves" of tree diagrams. "Grove" is the minimal gauge invariant class
- Applications
  - Processes with 6 fermions
  - Single W in e^+e^-
  - Single Z in e^+e^-
  - Simplification of quark flavour combinatorics in computations for hadron colliders
- Conclusions and Comments

- E.B. and T.Ohl Phys.Rev.Lett. 83 (1999) 480-483; E.B. and T.Ohl hep-ph9909487
- E.B. and M.Dubinin hep-ph/9909214, Talks given at the LCWS 99 and QFTHEP 99
- E.B., V.Ilyin et al. JHEP 0005 (2000) 052

The increase of the collider energies requires calculations with more and more particles in the final state and with better and better precision

- LEP1 - basically 2 fermion (γ) physics
- LEP2 - basically 4 fermion (γ) physics
- Tevatron, LC and LHC - in many cases 4, 5, 6 and 8 fermion physics with additional hard photons and/or gluons (jets)
  - top pair production with decays - 6 fermions;
  - single top production in W-gluon fusion mode - 5 fermions;
  - strongly interacting Higgs sector in hadronic collisions- $pp \to q\bar{q}W^+W^-$ - 6 fermions
  - study of Yukawa coupling - $pp(e^+e^-) \to t\bar{t}H$ - 8 fermions
  - etc
    - Large number of diagrams
    - Large number of subprocesses especially for LHC

One of the problem in process computations is a gauge cancellation among many of diagrams. Any method of calculation should preserve gauge invariance which is rather complicated in theories like SM with not a simple gauge group.

The well known statement from the quantum theory of gauge fields is that the whole set of Feynman diagrams contributing to any physics process is exactly gauge invariant.

In practical calculations it remains amazing how gauge cancellations take place

The question:

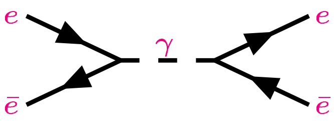
if there are exact gauge invariant subclasses of diagrams and how one can find them?

The answer is important in practice

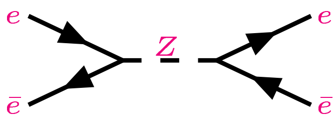
- any physics approximation should be based on gauge invariant classes of diagrams
- better precision of computations in many cases
- better understanding of physics parameters like running couplings or scales, like QCD scale. Often different part of the same process need in different values because different kinematical regions might be important
- any MC generator needs in well behaved and compact matrix elements and the gauge invariance gives that

The generation of gauge invariant subclasses could be implemented in Automatic Programs

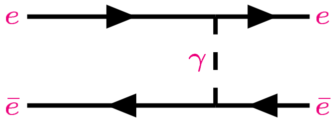
Bhabha-scattering



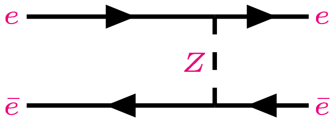
diagr.1



diagr.2



diagr.3



diagr.4

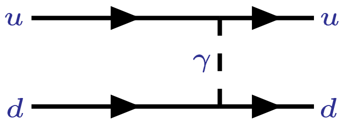
s-channel and t-channel are separately gauge invariant

It gives a simple example of the "fermion flip":

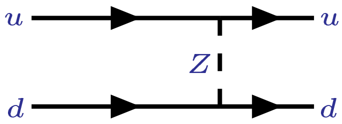
$e^+e^- \to \mu^+\mu^-$

$e^+\mu^- \to e^+\mu^-$

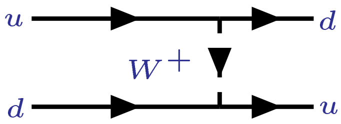
The less trivial example - u,d quarks scattering



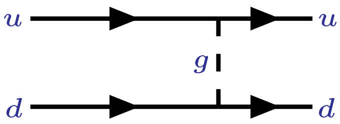
diagr.1



diagr.2



diagr.3



diagr.4

Three gauge invariant parts in this case:

- t-channel γ and Z exchanges
- t-channel W exchange
- QCD - t-channel gluon exchange

QCD part - obvious, it has an independent gauge group

One can make permitation to the process $ud \to cs$. Only the diagram with W exchange contributes.

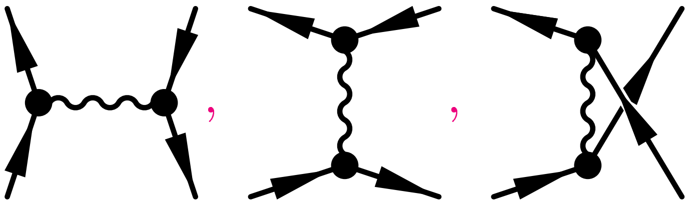
Several definitions.

Definition 1.

The Forest is a graph with each vertex representing a diagram and the edges given by the Flips of four-point subdiagrams.

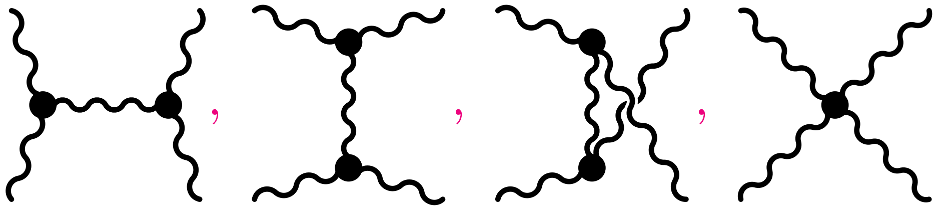
There are two kinds of flips: Guage and Flavor

Flavor Flip



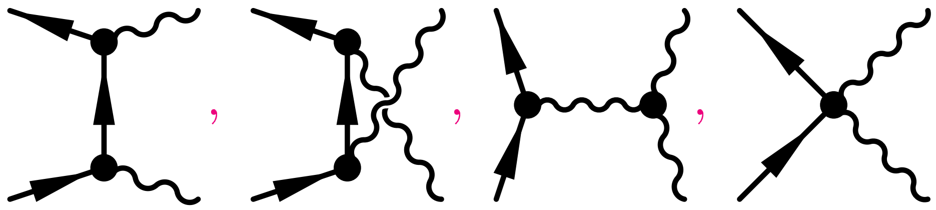
(1)

Gauge Flip



(2a)

and



(2b)

Definition 2.

The Gauge Forest is such a Forest or part of the Forest in which the points connected by the only gauge flips. The connected components of the Gauge Forest are called Groves

In general Groves form subgraphs of the Forest

Theorem.

The Forest F(E) for an external state E consisting of gauge and matter fields is connected if the fields in E carry no conserved quantum numbers other than the gauge charges. The Groves are the minimal gauge invariant classes of Feynman diagrams.

The proof is done by the induction method. [a]

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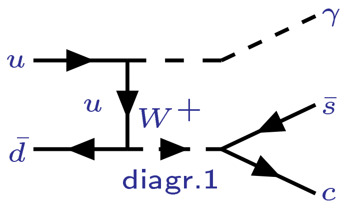
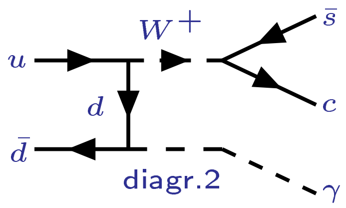
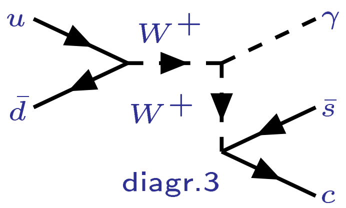
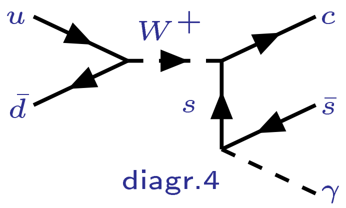
[a] E.B. and T.Ohl Phys.Rev.Lett. 83 (1999) 480-483; E.B. and T.Ohl hep-ph9909487

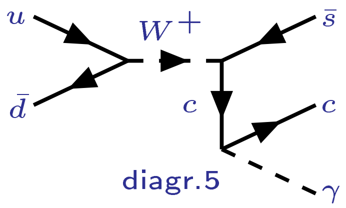
The process $u\bar{d} \to c\bar{s}\gamma$





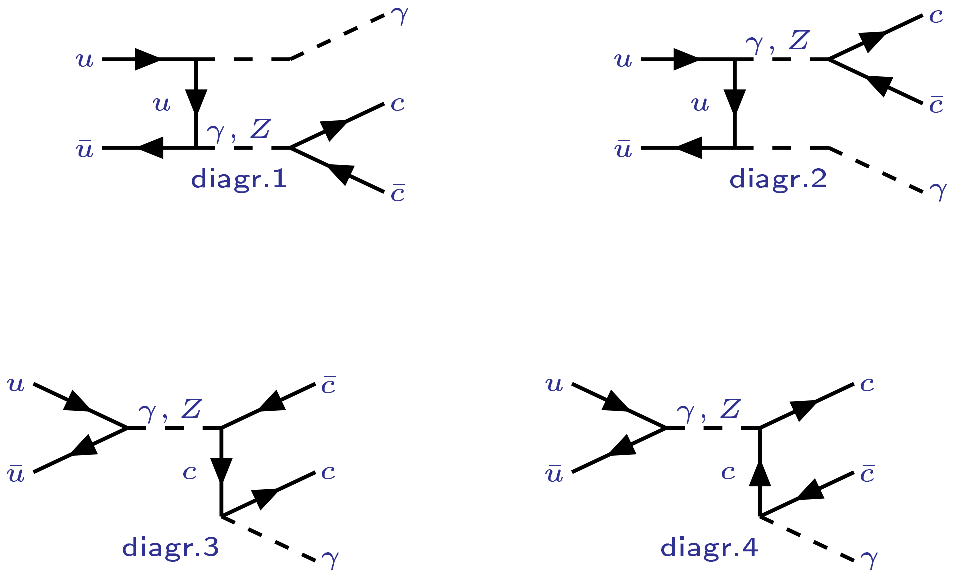






All diagrams are connected by the gauge flips passing through the middle diagram 3. Therefore all of them form only one Grove - one gauge invariant set of diagrams.

The process $u\bar{u} \to c\bar{c}\gamma$ (electroweak part, 8 diagrams)



Diagrams 1 and 2 related by the gauge flip, and diagrams 3 and 4 as well.

No gauge flip passes through the (γ, Z) line.

8 diagrams forms one disconnected forest which consists of two groves. So there are 2 gauge invariant subsets of 4 diagrams each.

In contrast to the previous case the initial and final state radiations are gauge independent here.

Groves for all processes with six massless fermions in the standard model (without QCD) [a]

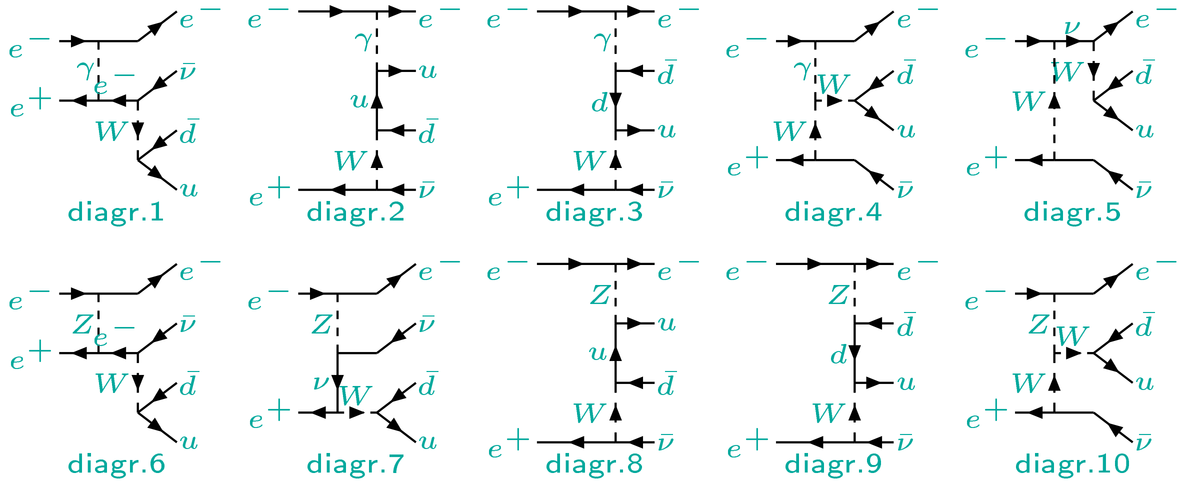
E	$\sum$	classes
$u\bar{u}u\bar{u}u\bar{u}$	144	$18 \cdot 8$
$u\bar{u}u\bar{u}u\bar{u}\gamma$	1008	$18 \cdot 24 + 36 \cdot 16$
$u\bar{u}u\bar{u}d\bar{d}$	92	$4 \cdot 11 + 6 \cdot 8$
$u\bar{u}u\bar{u}d\bar{d}\gamma$	716	$4 \cdot 95 + 6 \cdot 24 + 12 \cdot 16$
$\ell^+\ell^- u\bar{u}d\bar{d}$	35	$1 \cdot 11 + 3 \cdot 8$
$\ell^+\ell^- u\bar{u}d\bar{d}\gamma$	262	$1 \cdot 94 + 3 \cdot 24 + 6 \cdot 16$
$\ell^-\nu d\bar{u}d\bar{d}$	20	$2 \cdot 10$
$\ell^-\nu d\bar{u}d\bar{d}\gamma$	152	$2 \cdot 76$
$\ell^+\ell^-\ell^-\nu d\bar{u}$	20	$2 \cdot 10$
$\ell^+\ell^-\ell^-\nu d\bar{u}\gamma$	150	$2 \cdot 75$
$\ell^-\nu\ell^+\bar{\nu}d\bar{d}$	19	$1 \cdot 9 + 2 \cdot 4 + 1 \cdot 2$
$\ell^-\nu\ell^+\bar{\nu}d\bar{d}\gamma$	107	$1 \cdot 59 + 2 \cdot 12 + 2 \cdot 8 + 2 \cdot 4$
$\ell^-\bar{\nu}\ell^+\nu\ell^+\ell^-$	56	$4 \cdot 9 + 4 \cdot 4 + 2 \cdot 2$
$\ell^-\bar{\nu}\ell^+\nu\ell^+\ell^-\gamma$	328	$4 \cdot 58 + 4 \cdot 12 + 4 \cdot 8 + 4 \cdot 4$
$\ell^+\nu\ell^-\bar{\nu}\nu\bar{\nu}$	36	$4 \cdot 6 + 6 \cdot 2$
$\ell^+\nu\ell^-\bar{\nu}\nu\bar{\nu}\gamma$	132	$4 \cdot 26 + 2 \cdot 6 + 4 \cdot 4$
$\nu\bar{\nu}\nu\bar{\nu}\nu\bar{\nu}$	36	$18 \cdot 2$

includes the familiar LEP2 classes (CC09, CC10, CC11, etc.)

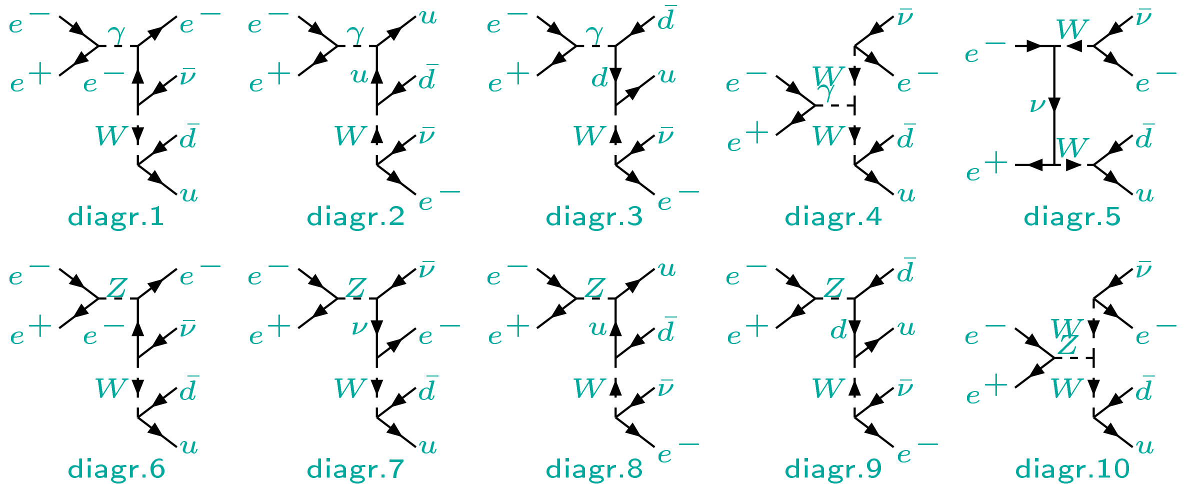
[a] T. Ohl, Objective Caml program *bocages* (1999, unpublished)

The CC20 process $e^+e^- \to e^-\bar{\nu}_e u\bar{d}$ contains two exact minimal gauge invariant classes ("groves") of diagrams: CC10 t-channel and CC10 s-channel

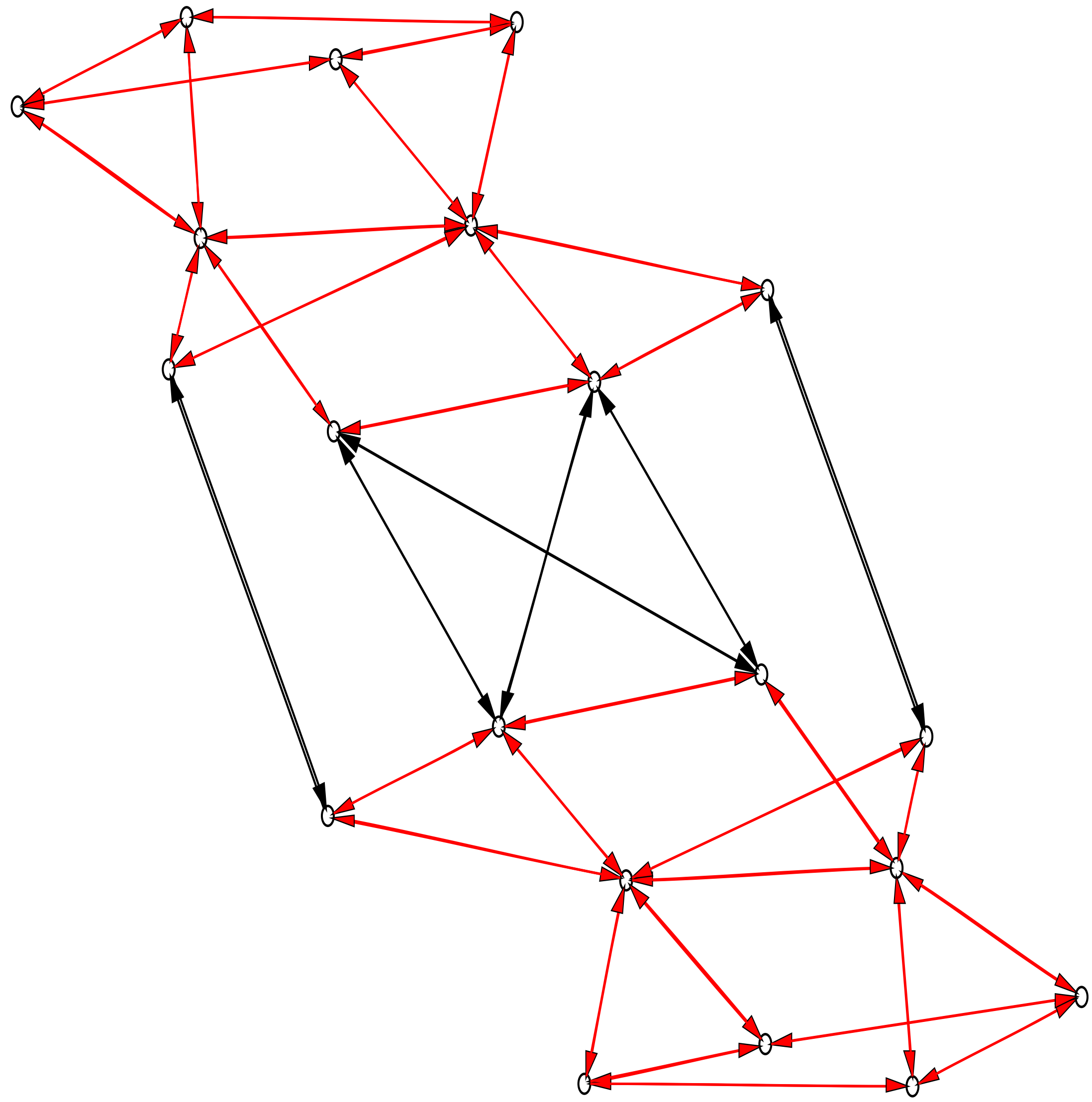
CC10 t-channel



CC10 s-channel



The Forest for the CC20 process contains two CC10 Groves (t- and s- channel)



[a]

Red lines - gauge flips
Black lines - flavour flips

[a] T. Ohl, Objective Caml program *bocages* (1999, unpublished)

Contributions to the total cross section (in fb) for the CC20 process at LEPII and LC energies

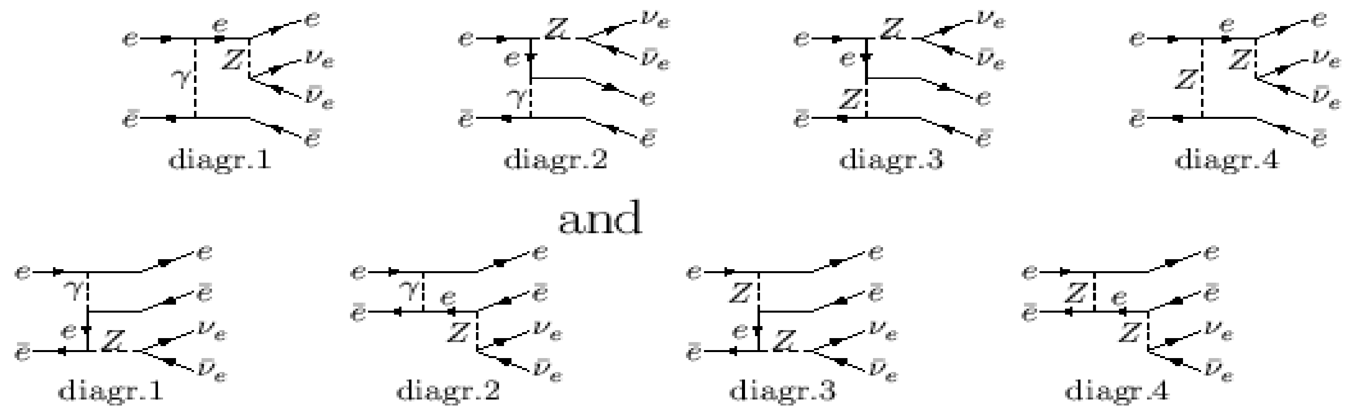
$\sqrt{s}$	$\sigma(CC10-t)$	$\sigma(CC10-s)$	$\sigma(t-s\ interf.)$	σ_{tot}
quark phase space cuts, no ISR				
183	130(0)	655(1)	0.2(0)	785(1)
190	147(0)	680(1)	5(0)	832(1)
350	635(1)	420(1)	21(0)	1076(1)
500	1127(2)	270(0)	19(0)	1417(2)
800	1981(4)	143(0)	16(0)	2140(4)
quark phase space cuts, with ISR				
183	117(0)	566(1)	0.2(0)	683(1)
190	132(0)	603(1)	5(0)	739(1)
350	587(1)	432(1)	20(0)	1039(1)
500	1049(5)	289(0)	19(0)	1357(5)
850	1873(4)	159(0)	16(1)	2048(4)
lepton and quark phase space cuts, no ISR				
183	102(0)	2(0)	0.0(0)	104(0)
190	116(0)	2(0)	0.0(0)	118(0)
350	513(1)	7(0)	0.2(0)	520(1)
500	928(2)	10(0)	0.3(0)	938(2)
800	1671(4)	15(0)	0.4(0)	1686(4)
lepton and quark phase space cuts, with ISR				
183	92(1)	2(0)	0.0(0)	94(1)
190	103(1)	2(0)	0.0(0)	105(1)
350	472(1)	6(0)	0.0(0)	478(1)
500	854(9)	10(0)	0.3(0)	864(9)
800	1581(3)	15(0)	0.4(0)	1596(3)

quark phase space cuts: $E_q \geq 3$ GeV, $M_{ud} \geq 5$ GeV

lepton phase space cut: $\cos\theta_e \geq 0.997$

Single Z production

The process $e^+e^- \to e^+e^-\nu\bar{\nu}$ (56 Feynman Diagrams)



Two classes of 4 diagrams each contribute to single Z

In total 10 Gauge Ivariant Classes or 10 Groves

56 D = 4 * 9D + 4 * 4D + 2 * 2D

Contributions of the gauge invariant subsets (fb) at the energy $\sqrt{s}$ =200 GeV. First row - with angular cuts, second row - no angular cuts for e^-, e^+.

Angular cuts: $\cos\theta_{e^-} \geq$ 0.997 and $\cos\theta_{e^+} \leq$ 0.997
Lepton energy cut: $E_l \geq$ 15 GeV

$8Z$ - single Z-production
$18W$ - single W^+ and W^- (2*CC09)

	$18W$	$8Z$	$9W^+W^-$	$4ZZ$
θ_e, E_l	36.1	16.4	0.91	0.02
only E_l	106.6	153.6	240.5	44.9

	$9\nu_e\bar{\nu}_e$	$4e^+e^-$	$2\nu_e\bar{\nu}_e$	2 t-ch
θ_e, E_l	$8{\cdot}10^{-3}$	$7{\cdot}10^{-5}$	$1{\cdot}10^{-5}$	$6{\cdot}10^{-7}$
only E_l	15.9	0.02	$3{\cdot}10^{-3}$	$8{\cdot}10^{-4}$

If the energy and angular cuts are applied still there are significant contributions from both single W and Z productions. One should be careful in interpretation of experimental measurements in this channel.

Simplification of Flavour Combinatorics in the Evaluation of Hadronic Processes

A serious computational problem is the large number of partonic subprocesses (Some of them are very much similar)

- many quark partons with different flavors
- many additional diagrams for each subprocess because of CKM quark mixing

Two approximations:

- $$V_{CKM} \Longrightarrow \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}, \qquad V = \begin{pmatrix} \cos\vartheta_c & \sin\vartheta_c \\ -\sin\vartheta_c & \cos\vartheta_c \end{pmatrix}$$

  where ϑ_c is the Cabbibo angle.
- $M_u = M_d = M_s = M_c = 0$

Basic ideas:

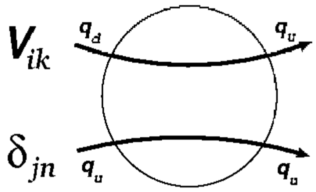
- rotation of down quarks, thus, transporting the mixing matrix elements from vertices of subprocess Feynman diagrams to the parton distribution functions
- diagrams are divided into gauge invariant classes with different topology of the quark line

New rules for convolution with structure functions depend on the topology of the gauge invariant class [a]

[a] E.B., V.Ilyin et al. JHEP 0005 (2000) 052, realized in CompHEP version for hadron collisions V41.10

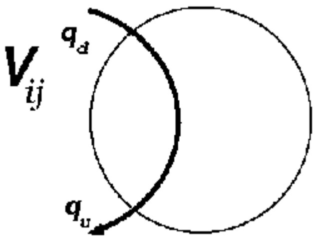
Examples

Scattering topology. 1st Rule



$$|\mathcal{D}_{sc}|^2 \implies \int dx_1 dx_2\,[f_d(x_1)+f_s(x_1)]\,[f_{\bar{u}}(x_2)+f_{\bar{c}}(x_2)]\,|\mathcal{M}|^2$$

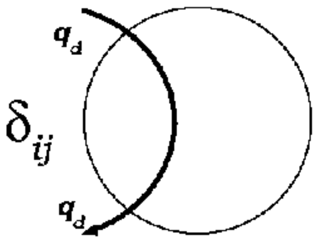
Annihilation CC topology. 2nd Rule



$$|\mathcal{D}_a^{CC}|^2 \implies \int dx_1 dx_2 [f_d(x_1)\,f_{\bar{u}}(x_2)\,\cos^2\vartheta_c + f_s(x_1)\,f_{\bar{c}}(x_2)\,\cos^2\vartheta_c +$$

$$+ f_d(x_1)\,f_{\bar{c}}(x_2)\,\sin^2\vartheta_c + f_s(x_1)\,f_{\bar{u}}(x_2)\,\sin^2\vartheta_c]|\mathcal{M}|^2$$

Annihilation NC topology. 3d Rule



$$|\mathcal{D}_a^{NC}|^2 \implies \int dx_1 dx_2\,[f_d(x_1)\,f_{\bar{d}}(x_2) + f_s(x_1)\,f_{\bar{s}}(x_2)]\,|\mathcal{M}|^2$$

The Exchange topology and all possible interferences between the above topologies lead to the same rules

Conclusions and Comments

- We have discussed the method which allows to split the complete set of Feynman diagrams contributed to a physics process on gauge invariant subclasses. Well known gauge invariant classes of diagrams like CC10, CC11, CC09 etc naturally appear in such an approach. It was demonstrated the above classes are minimal invariant classes ("Groves").
- For a concrete physical process one creates the graph - "Forest" in which vertices represent diagrams and edges show the connection between diagrams by possible flips, flavor and gauge. The vertices (diagrams) connected by the only gauge flips form connected subgraphs, "Groves", and the corresponding diagrams form minimal gauge invariant classes. The flavor flips connect diagrams from different gauge invariant classes.
- Separation on gauge invariant classes in some cases allows to understand better properties of processes, to get better precision of calculations, to make in tree level computations a natural choice of characteristic scales for ISR, structure functions, running couplings etc, to get reasonable approximations, it leads to a simplification of quark flavor combinatorics
- In some cases for processes with multiparticle final states the number of gauge invariant classes is much smaller than the number of physical reactions. So one can compute amplitudes for gauge invariant subclasses of diagrams and then compute processes by taken different combinations of that amplitudes.
- The analysis was done for the tree level Feynman diagrams. A consideration at loop level is in progress.