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Calculation of three-loop QCD diagrams for massive baryonic correlators

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We present analytical next-to-leading order results for the correlator of baryonic currents at the three-loop level with one finite mass quark. We obtain the massless and the HQET limits of the correlator as particular cases from the general formula, we also give explicit expressions for the moments of the spectral density. Calculations have been performed with an extensive use of the symbolic manipulation programs MATHEMATICA and REDUCE.

Publications

S. Groote, J.G. Körner and A.A. Pivovarov, Phys. Rev. **D61** (2000) 071501;
S. Groote, J.G. Körner and A.A. Pivovarov, hep-ph/0009218, prepared for the proceedings

Physical motivation

- heavy baryons have been studied experimentally with high precision

- prominent object of the QCD analysis is the correlator of two baryonic currents. This object allows for a QCD sum rule analysis in order to determine basic quantities like the ground state energy

- theoretical calculations beyond the leading order have not been done for many interesting cases up to now

Currents and correlators

Baryonic currents with two massless quarks u and d and one massive quark q are modelled as composite operators of the quark fields.

$$j = \epsilon^{abc}(u_a^T C d_b)\Gamma\Psi_c$$

Ψ: quark field with mass m

C: charge conjugation matrix

T: transposition

ϵ^{abc}: total antisymmetric tensor

a, b, c: color indices of $SU(3)$

Γ: current-type matrix with

$\rightarrow$ $\Gamma = 1$: scalar baryon

$\rightarrow$ $\Gamma = \gamma_5$: pseudoscalar baryon

The correlator is given by a momentum part $\Pi_q(q^2)$ and a mass part $\Pi_m(q^2)$,

$$i\int \langle Tj(x)\bar{j}(0)\rangle e^{iqx} = \gamma_\nu q^\nu \Pi_q(q^2) + m\Pi_m(q^2)$$

The spectral density

The spectral density $\rho^{\#}(s)$ is the real object of interest for phenomenological applications.

$$\Pi_{\#}(q^2) = \frac{1}{128\pi^4}\int_{m^2}^{\infty}\frac{\rho^{\#}(s)ds}{s-q^2}$$

The spectral density can be calculated as discontinuity of the correlator which has a cut along the negative real axis.

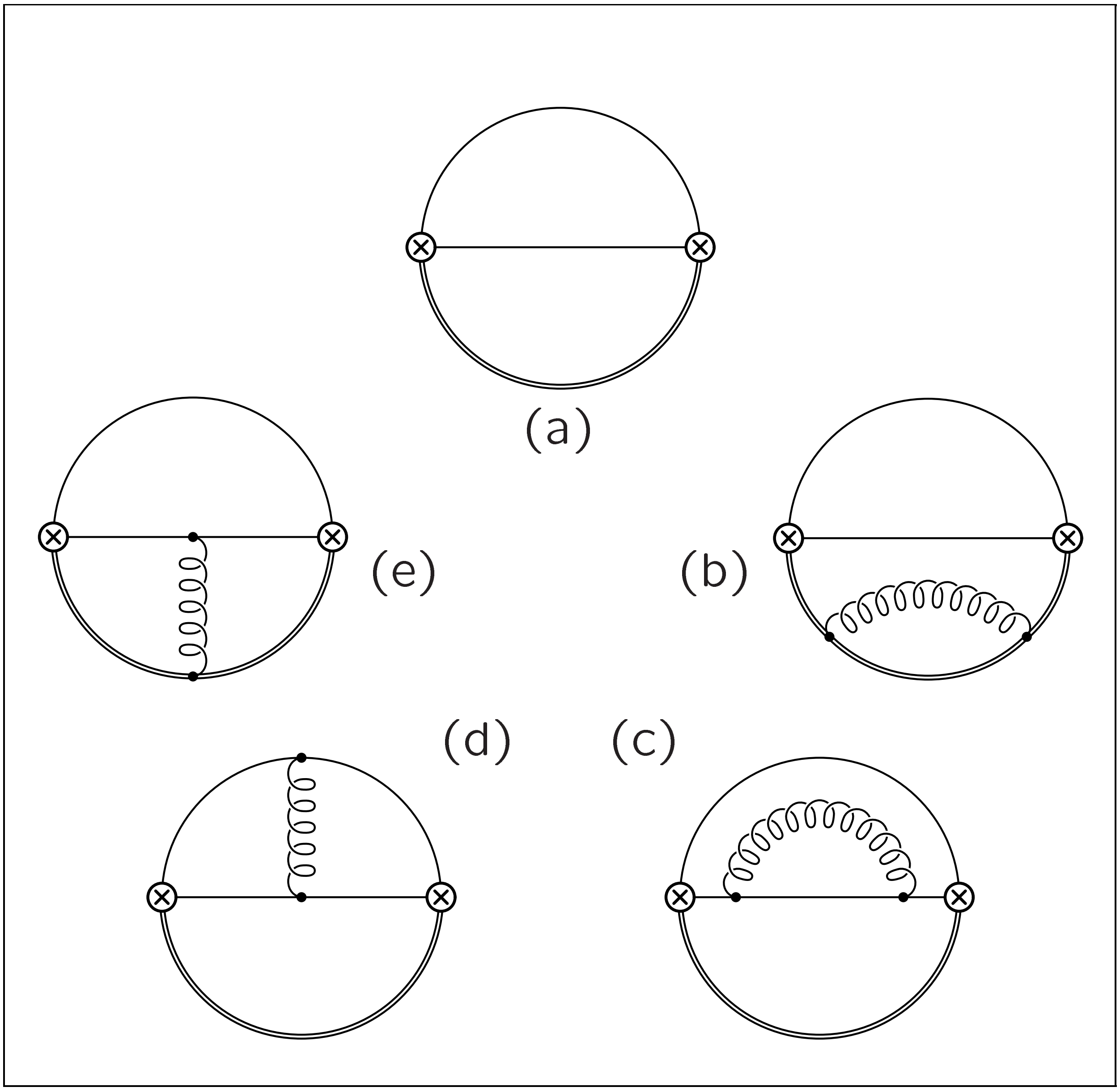
$$\rho^{\#}(s) = \frac{1}{2\pi i}\left(\Pi_{\#}(se^{-i\pi}) - \Pi_{\#}(se^{i\pi})\right)$$

Task is to calculate leading and next-to-leading order contributions to the spectral density.

$$\rho^{\#}(s) = s^2\left\{\left(1+\frac{\alpha_s}{\pi}\ln\left(\frac{\mu^2}{m^2}\right)\right)\rho_0^{\#} + \frac{\alpha_s}{\pi}\rho_1^{\#}\right\}$$

Contributing diagrams

The following correlator diagrams had to be calculated in QCD. The double line indicates the massive quark, the spring is the gluon.



Calculation methods

1. all diagrams reduce to scalar prototypes

2. integration over massless loops is done by using recurrence relations

3. we are left with

$$V(\alpha, \beta; q^2/m^2) = \int \frac{d^D k}{(k^2 + m^2)^\alpha (q - k)^{2\beta}}$$

4. in case of diagram (e):

   the basic integral enters as a subdiagram depending on the internal loop momentum

5. a finite range (convolution type) integration over this internal momentum with a spectral density of the basic integral V

Calculation tools

- Intel P-II 400MHz (64MB memory) under Debian Linux 2.1

- REDUCE and MATHEMATICA for the calculation of massless contribution

- MATHEMATICA for the calculation of prototypes

- MATHEMATICA for the collection and composition of parts

- MATHEMATICA for the calculation of moments

The calculation of the massive contributions had to be done by hand because they do not follow a fixed algorithm. Future developments will automatize these calculations as well.

Results for the mass part

→ Leading order result ($z = m^2/s$)

$$\rho_0^m = 1 + 9z - 9z^2 - z^3 + 6z(1+z)\ln z$$

→ Next-to-leading order result

$$\begin{aligned}
\rho_1^m = \; & 9 + \frac{665}{9}z - \frac{665}{9}z^2 - 9z^3 \\
& - \left(\frac{58}{9} + 42z - 42z^2 - \frac{58}{9}z^3\right)\ln(1-z) \\
& + \left(2 + \frac{154}{3}z - \frac{22}{3}z^2 - \frac{58}{9}z^3\right)\ln z \\
& + 4\left(\frac{1}{3} + 3z - 3z^2 - \frac{1}{3}z^3\right)\ln(1-z)\ln z \\
& + 12z\left(2 + 3z + \frac{1}{9}z^2\right)\left(\frac{1}{2}\ln^2 z - \zeta(2)\right) \\
& + 4\left(\frac{2}{3} + 12z + 3z^2 - \frac{1}{3}z^3\right)\mathrm{Li}_2(z) \\
& + 24z(1+z)\left(\mathrm{Li}_3(z) - \zeta(3) - \frac{1}{3}\mathrm{Li}_2(z)\ln z\right)
\end{aligned}$$

Results for the momentum part

→ Leading order result ($z = m^2/s$)

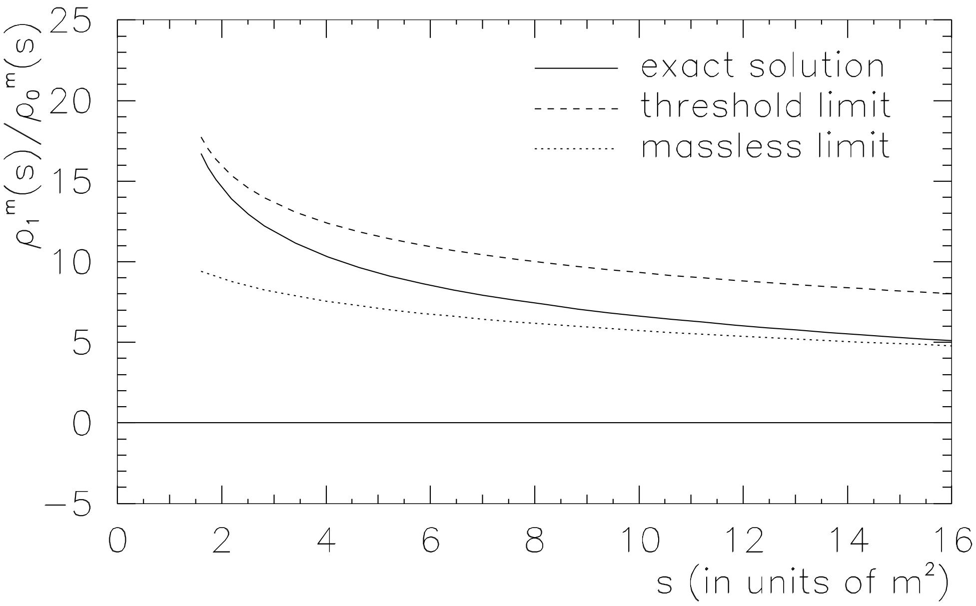
$$\rho_0^q = \frac{1}{4} - 2z + 2z^3 - \frac{1}{4}z^4 - 3z^2 \ln z$$

→ Next-to-leading order result

$$\begin{aligned}
\rho_1^q = {} & \frac{71}{48} - \frac{565}{36}z - \frac{7}{8}z^2 + \frac{625}{36}z^3 - \frac{109}{48}z^4 \\
& - \left(\frac{49}{36} - \frac{116}{9}z + \frac{116}{9}z^3 - \frac{49}{36}z^4\right) \ln(1-z) \\
& + \left(\frac{1}{4} - \frac{17}{3}z - 11z^2 + \frac{113}{9}z^3 - \frac{49}{36}z^4\right) \ln z \\
& + \left(\frac{1}{3} - \frac{8}{3}z + \frac{8}{3}z^3 - \frac{1}{3}z^4\right) \ln(1-z) \ln z \\
& -2z^2 \left(9 + \frac{4}{3}z - \frac{1}{6}z^2\right) \left(\frac{1}{2}\ln^2 z - \zeta(2)\right) \\
& + \left(\frac{2}{3} - \frac{16}{3}z - 18z^2 + \frac{8}{3}z^3 - \frac{1}{3}z^4\right) \mathrm{Li}_2(z) \\
& -12z^2 \left(\mathrm{Li}_3(z) - \zeta(3) - \frac{1}{3}\mathrm{Li}_2(z)\ln(z)\right)
\end{aligned}$$

Physical result I: going to different limits

The massless limit $m \to 0$ and the threshold limit can be easily obtained as limiting cases of our expressions. For the threshold limit we get even further than recent HQET calculations.



The figure shows the ratio $\rho_1^m(s)/\rho_0^m(s)$ in dependence of the energy square s.

On the other hand, our exact curve interpolates nicely between the two limits. This might be essential for higher loop order estimates.

Physical result II: calculating the moments

$$\mathcal{M}_n^{\#} = \int_{m^2}^{\infty} \frac{\rho^{\#}(s)ds}{s^n} = m^{6-2n} M_n^{\#}$$
$$M_n^{\#} = M_n^{\#(0)} \left\{ 1 + \frac{\alpha_s}{\pi} \left(\ln\left(\frac{\mu^2}{m^2}\right) + \delta_n^{\#} \right) \right\}$$

$$M_n^{q(0)} = \frac{12}{(n+1)n(n-1)^2(n-2)(n-3)},$$
$$M_n^{m(0)} = \frac{12}{n(n-1)^2(n-2)^2(n-3)},$$

In contrast to the previous one, the quantity

$$\frac{M_n^{\#}}{M_n^{\#(0)}} = \frac{M_N^{\#}}{M_N^{\#(0)}} \left\{ 1 + \frac{\alpha_s}{\pi}(\delta_n^{\#} - \delta_N^{\#}) \right\}$$

(for fixed N) is a renormalization scheme independent quantity because $(\delta_n^{\#} - \delta_N^{\#})$ is scheme independent. These differences can be calculated automatically using MATHEMATICA.

$$\delta_n^{\#} = A_n^{\#} + 2\pi^2/9$$

n	A_n^m	$\delta_n^m - \delta_{n-1}^m$
4	3	
5	13/2	3.500000
6	17/2	2.000000
7	535/54	1.407407
8	1187/108	1.083333
9	64093/5400	0.878333
10	22691/1800	0.737037
11	1167767/88200	0.633878
12	2433499/176400	0.555357

n	A_n^q	$\delta_n^q - \delta_{n-1}^q$
4	9/2	
5	22/3	2.833333
6	109/12	1.750000
7	5593/540	1.274074
8	6133/540	1.000000
9	460351/37800	0.821190
10	40553/3150	0.695370
11	148574/11025	0.602132
12	2470739/176400	0.530357