Feynman-diagram evaluation in the electroweak theory with computer algebra

Georg Weiglein, CERN

Fermilab, 10/2000

1. Introduction
2. Perturbative evaluation of gauge theories with computer algebra
3. Examples of higher-order results in the SM and the MSSM
4. Application of computer-algebra systems beyond tools for calculations
5. Conclusions

1. Introduction

Computer algebra (CA):

Exact algebraic calculations: no round-off errors, symbolic manipulations, analytical results

Development of CA systems boosted by applications in particle physics

Schoonschip
[*M. Veltman 1967*] *Nobel Prize '99*

Further CA systems with roots in particle physics:

Macsyma [*J. Moses '74*]

Reduce [*A. Hearn '73*]

Mathematica [*S. Wolfram '88*]

FORM [*J. Vermaseren '91*] ...

CA systems:

- built-in algorithms
- framework for user-implemented algorithms
- communication with external programs:
  numerics, text processing, graphics, ... ,
  other CA systems

Examples: *Maple*, *Mathematica* and *FORM*

Maple:

[*B. Char, K. Geddes, M. Gentleman, G. Gonnet '83*]

[*University of Waterloo*]

own programming language, open source code

Mathematica:

[*S. Wolfram '88*]

programmed in C, source code not available

General features of *Maple* and *Mathematica*:

– general purpose programs

– many built-in functions

– symbolic and numerical calculations, graphics, sound

– many software packages available

– user-friendly, interactive platform

– not optimized for specific problems

– non-local operations:
  need all terms of an expressions inside memory
  of computer, e.g.: factorization
  $\Rightarrow$ limited mainly by physical memory

FORM:

[*J. Vermaseren '91*]

– optimized for very large expressions

– expressions are fully expanded
  operations carried out locally on single terms
  only non-local operation: normal ordering
  ⇒ limited mainly by disk space

– less user-friendly, non-interactive execution

– few built-in operations

FORM parallelization project:

[*D. Fliegner, A. Rétey, J. Vermaseren '99*]

[*DFG Forschergruppe, Karlsruhe*]

input terms distributed over several processors

2. Perturbative evaluation of gauge theories with computer algebra

Standard Model (SM):

Electroweak interaction:

gauge bosons: γ, Z, W^+, W^-

fermions: 3 generations of quarks and leptons

$$\begin{array}{ccc} u, d, & s, c & t, b \\ \nu_e, e & \nu_\mu, \mu & \nu_\tau, \tau \end{array}$$

Masses of gauge bosons and fermions via Higgs mechanism: $\Rightarrow$ scalar field H

Last missing ingredient of the SM

Strong interaction: QCD

gauge bosons: $g_1, \ldots g_8$: gluons

quarks: q_{r}, q_{b}, q_{g}

Best motivated extension of SM: Supersymmetry

symmetry between fermions and bosons

$$\begin{aligned} Q|\text{boson}\rangle &= |\text{fermion}\rangle \\ Q|\text{fermion}\rangle &= |\text{boson}\rangle \end{aligned}$$

Solution for hierarchy problem, gauge coupling unification, dark matter, etc.

Minimal Supersymmetric Standard Model (MSSM):

Superpartners for Standard Model particles

$$\begin{array}{llll} \Big[u,d,c,s,t,b\Big]_{L,R} & \Big[e,\mu,\tau\Big]_{L,R} & \Big[\nu_{e,\mu,\tau}\Big]_L & \text{Spin } \tfrac{1}{2} \\ \Big[\tilde{u},\tilde{d},\tilde{c},\tilde{s},\tilde{t},\tilde{b}\Big]_{L,R} & \Big[\tilde{e},\tilde{\mu},\tilde{\tau}\Big]_{L,R} & \Big[\tilde{\nu}_{e,\mu,\tau}\Big]_L & \text{Spin } 0 \end{array}$$

$$\begin{array}{llll} g & \underbrace{W^{\pm}, H^{\pm}} & \underbrace{\gamma, Z, H_1^0, H_2^0} & \text{Spin 1 / Spin 0} \\ \tilde{g} & \tilde{\chi}^{\pm}_{1,2} & \tilde{\chi}^0_{1,2,3,4} & \text{Spin } \frac{1}{2} \end{array}$$

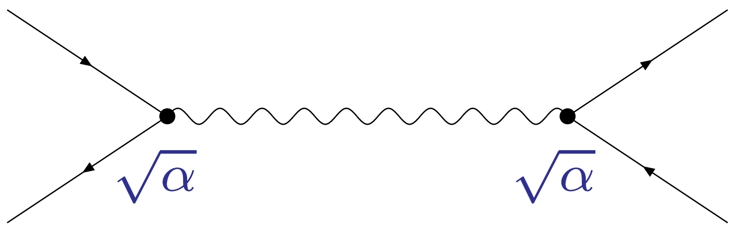
Enlarged Higgs sector:

Two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

Perturbation theory:

Interaction as perturbation to free field theory

⇒ Feynman diagrams

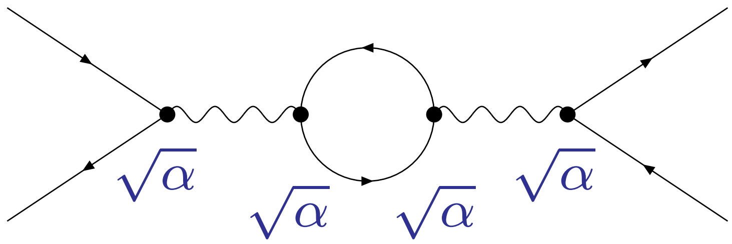


$\alpha \approx \frac{1}{137}$

expansion in coupling constant:

lowest order: classical limit

Quantum corrections: loop diagrams



⇒ integration over loop momenta

⇒ divergences

Gauge invariance ⇒ **theory is renormalizable**

[G. 't Hooft '71]

[G. 't Hooft, M. Veltman '72]

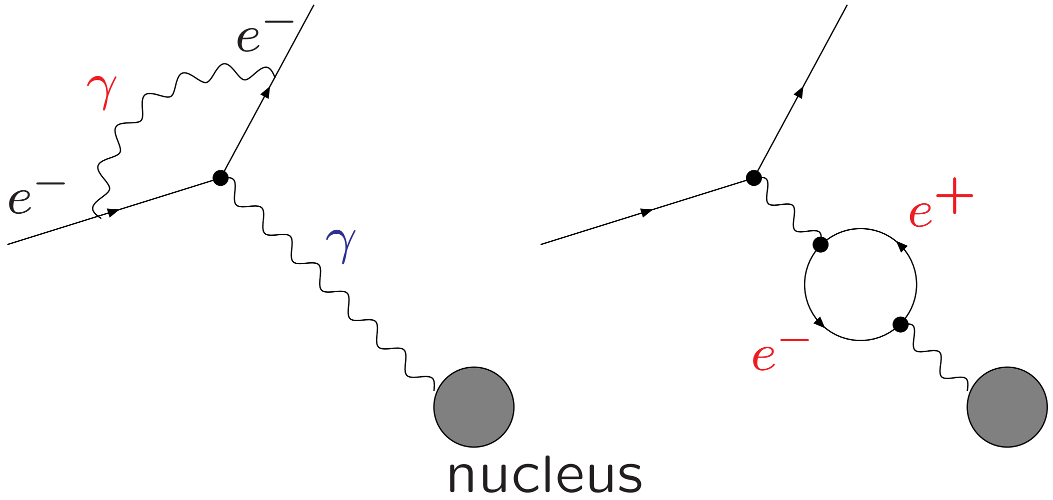
Nobel Prize '99

⇒ quantum effects consistently calculable

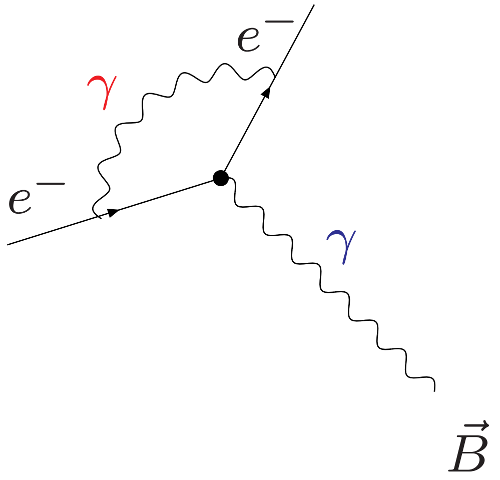
Precision tests of the electroweak theory:

Historical: QED

Lamb shift:



$g-2$:



$$a = \frac{1}{2}(g-2)$$

$$\begin{aligned} a_{\text{exp}} &= 1\,159\,652\,188(\pm 4) \times 10^{-12} \\ a_{\text{theo}} &= 1\,159\,652\,157(\pm 28) \times 10^{-12} \end{aligned}$$

Electroweak precision measurements:

- LEP1/SLC: e^+e^- annihilation at the Z-boson resonance
  LEP1: $\sim 4\times 10^6$ events/exp. ('89 – '95)
- LEP2: $e^+e^- \to W^+W^-$
- Tevatron: $q\bar{q}' \to W \to l\bar{\nu},\ q\bar{q}'$
  $q\bar{q} \to t\bar{t},\ t \to W^+ b \to \ldots$
- low-energy experiments

$M_{\mathrm{Z}}[\,\mathrm{GeV}]$	$=$	91.1875 ± 0.0021	0.002%
$\Gamma_{\mathrm{Z}}[\,\mathrm{GeV}]$	$=$	2.4952 ± 0.0023	0.09%
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$	$=$	0.23147 ± 0.00017	0.07%
$M_{\mathrm{W}}[\,\mathrm{GeV}]$	$=$	80.434 ± 0.037	0.05%
$m_{\mathrm{t}}[\,\mathrm{GeV}]$	$=$	174.3 ± 5.1	2.9%
$G_\mu[\,\mathrm{GeV}^{-2}]$	$=$	$1.16637(1)\,10^{-5}$	0.0009%

Necessary to take into account at LEP:
tides, water level of Lake Léman,
train schedule TGV Geneva – Paris

Quantum effects of the theory:

$\sim \mathcal{O}(1\%)$: loop corrections

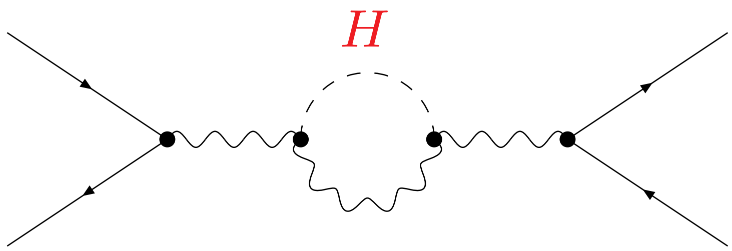
Comparison of ew precision data with theory:

EW precision data:	Theory:
$M_{\mathsf{Z}}, M_{\mathsf{W}}, \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}, \ldots$	SM, MSSM, ...

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



All parameters of the model enter

$\Downarrow$

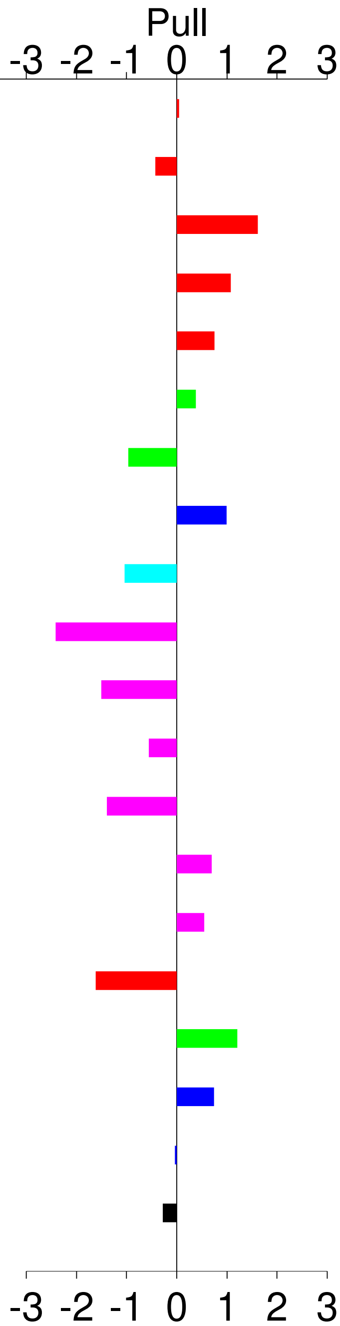
Improve indirect constraints on unknown parameters: M_{H}, $m_{\tilde{\mathsf{t}}}$, ...

effects of "new physics"?

Comparison of SM predictions with precision data:

Osaka 2000

	Measurement	Pull
m_Z [GeV]	91.1875 ± 0.0021	.05
Γ_Z [GeV]	2.4952 ± 0.0023	-.43
σ^0_{hadr} [nb]	41.540 ± 0.037	1.62
R_l	20.767 ± 0.025	1.08
$A^{0,l}_{fb}$	0.01714 ± 0.00095	.76
A_e	0.1498 ± 0.0048	.38
A_τ	0.1439 ± 0.0042	-.97
R_b	0.21649 ± 0.00072	1.00
R_c	0.1683 ± 0.0038	-1.04
$A^{0,b}_{fb}$	0.0989 ± 0.0020	-2.42
$A^{0,c}_{fb}$	0.0689 ± 0.0035	-1.50
A_b	0.922 ± 0.023	-.55
A_c	0.632 ± 0.026	-1.40
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.70
m_W [GeV]	80.427 ± 0.046	.55
$\sin^2\theta^{lept}_{eff}$	0.23098 ± 0.00026	-1.62
$\sin^2\theta_W$	0.2255 ± 0.0021	1.20
m_W [GeV]	80.448 ± 0.062	.75
m_t [GeV]	174.3 ± 5.1	-.04
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02804 ± 0.00065	-.28



$\Rightarrow$ SM in good overall agreement with data

Indirect determination of m_t from precision data:

$$m_t = 172^{+14}_{-11}\ \text{GeV}$$

direct measurement:

$$m_t = 174.3 \pm 5.1\ \text{GeV}$$

One-loop corrections to precision observables:

$$\sim m_t^2$$

$$\sim \ln M_H$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

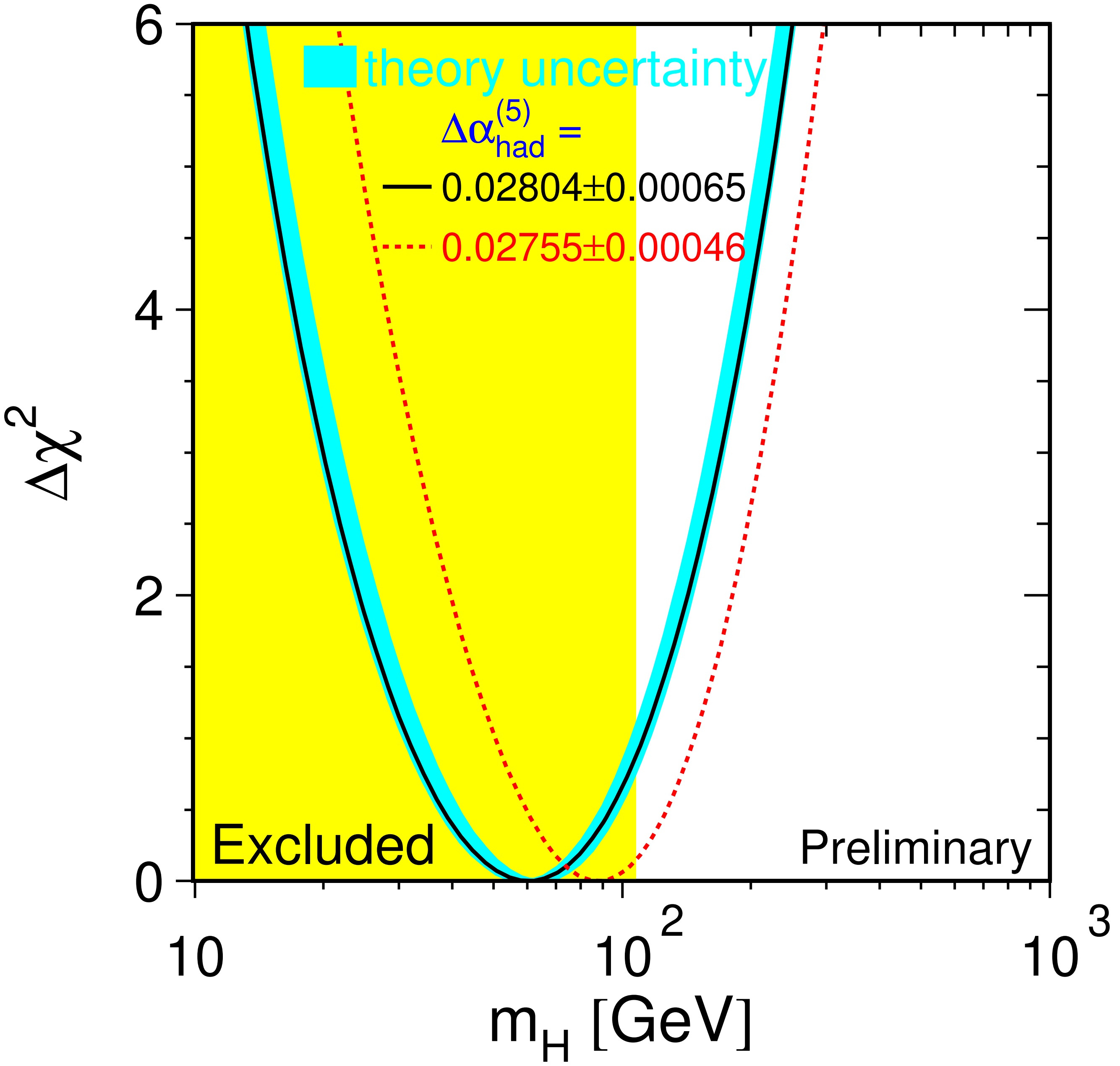
Theoretical uncertainties:

– unknown higher-order corrections

– exp. error of input parameters: m_t, $\Delta\alpha_{had}$, ...

Global fit to all data in the SM:

[*LEPEWWG '00*]



$\Rightarrow M_{\mathrm{H}} < 170$ GeV, 95% C.L.

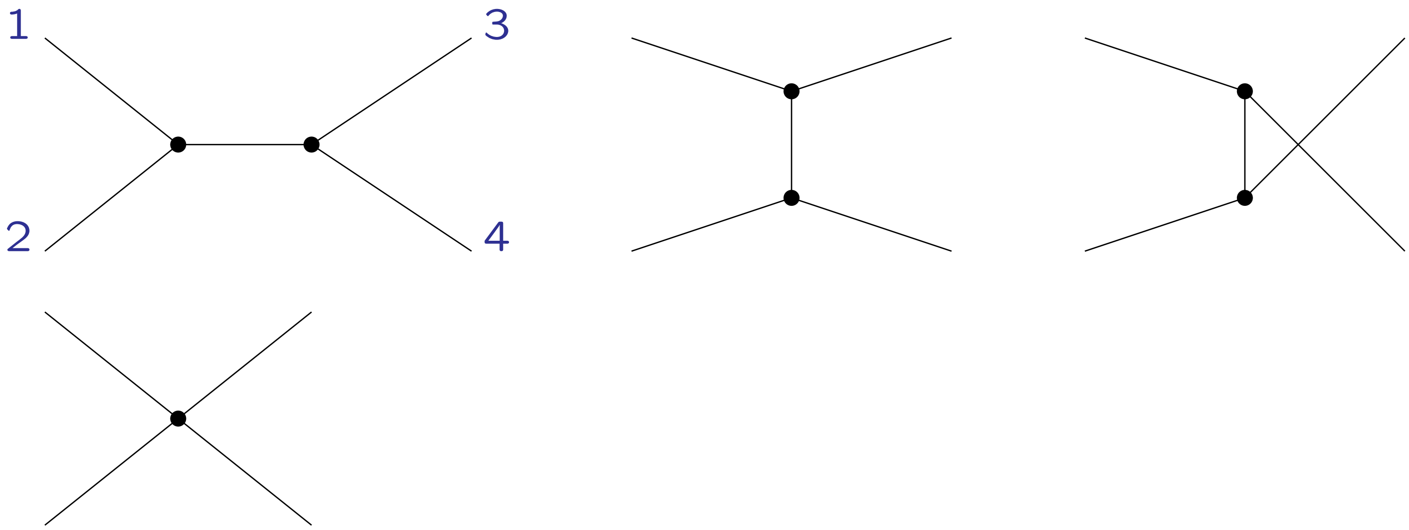
Evaluation of Feynman diagrams

Algorithmic procedure:

Topology creation:

Specified number of loops and external legs
$\Rightarrow$ generation of all topologically different diagrams (in renormalizable models: only 3-point and 4-point vertices possible)

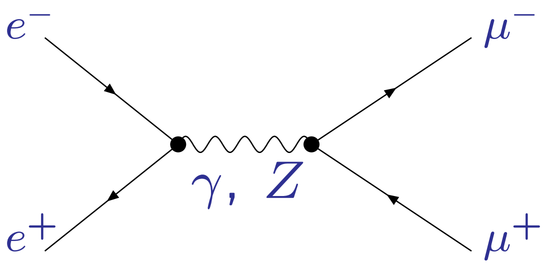
Example: $2 \to 2$, tree level



Field insertion:

insertion of fields of specified model into topologies in all possible ways $\Rightarrow$ Feynman diagrams

Example: $e^-e^+ \to \mu^-\mu^+$, tree level



Feynman rules:

$\Rightarrow$ mathematical expression

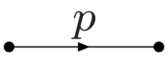
Example: Feynman rules of QED

- photon propagator (Feynman gauge):

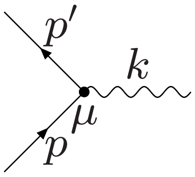
$$\Delta_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} =$$

- electron/positron propagator:

$$S_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon} =$$



- electron–positron–photon vertex:



$$= (2\pi)^4 \delta^4(p - p' - k)(ie\gamma_\mu)$$

- ingoing and outgoing lines:

electron:		$= u(p,\sigma)$		$= \overline{u}(p,\sigma)$
Positron:		$= \overline{v}(p,\sigma)$		$= v(p,\sigma)$
Photon:		$= \epsilon_\mu(p,\lambda)$		$= \epsilon^*_\mu(p,\lambda)$

- integration over momenta of internal particles

  δ-distr. $\Rightarrow$ momentum conservation

  for loop diagrams $\Rightarrow \int \frac{d^4q}{(2\pi)^4}$ for each loop

- minus sign for fermion loops

- combinatorial factor

Regularization:

SM: DREG [*G. 't Hooft, M. Veltman '72*]

MSSM: DRED [*W. Siegel '79*], [*D. Capper et al. '80*]

Renormalization:

fixes physical meaning of parameters order by order

bare $\longrightarrow$ renormalized parameters:

$$e_0 = Z_e\, e = (1 + \delta Z_e)e, \quad m_0 = m + \delta m$$

δZ_e, δm: counterterms $\Rightarrow$ counterterm diagrams

renormalizable theory:
divergencies compensated by counterterms

Dirac algebra, Dirac traces:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad \Rightarrow \mathrm{Tr}\,\{\gamma_\mu\gamma_\nu\} = 4g_{\mu\nu}$$

Tensor-integral decomposition:

$$\int \frac{d^Dq}{(2\pi)^D} \frac{q_\mu}{(q^2 - m^2)\,[(q+k)^2 - m^2]} = a\; k_\mu$$

[*G. Passarino, M. Veltman '79*]

integration by parts

[*F. Tkachov '81*] [*K. Chetyrkin, F. Tkachov '81*]

$\Rightarrow$ scalar integrals:

$$A_0(m^2) = \frac{1}{i\pi^2}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{q^2-m^2}$$
$$B_0(p^2,m_1^2,m_2^2) = \frac{1}{i\pi^2}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{(q^2-m_1^2)\left[(q+k)^2-m_2^2\right]}$$

Evaluation of standard scalar integrals:

One-loop:

reducible to A_0, B_0, C_0, D_0:

$$A_0, B_0 : \log(\sqrt{\ldots}); \quad C_0, D_0 : \log,\ \mathrm{Li}_2(x) = \int_0^1 dt \frac{\ln t}{t-1/x}$$

Two-loop:

in general not expressible in terms of polylogarithmic functions

[*R. Scharf '91*]

$\Rightarrow$ – numerical evaluation

– asymptotic expansions

On-shell conditions for external legs

$\Rightarrow$ S-Matrix elements

$|\mathcal{M}^2|$, phase-space integration

$\Rightarrow$ cross sections, life times

Application of computer algebra programs:

- result for arbitrary parameters; insertion of numerical values only in final step

- algebraic result

  $\Rightarrow$ highly non-trivial checks:

  UV-, IR finiteness

  gauge-parameter independence

  Ward identities (symmetry relations)

  directly at algebraic level

  e.g. SM, 2-loop order:

$$p^2\left[\tilde{\Sigma}_{\mathrm{L}}^{ZZ,(2)} - 2iM_{\mathrm{Z}}\tilde{\Sigma}^{Z\chi,(2)}\right] - M_{\mathrm{Z}}^2\tilde{\Sigma}^{\chi\chi,(2)}$$
$$- p^2\left(\tilde{\Sigma}^{Z\chi,(1)}\right)^2 + \tilde{\Sigma}_{\mathrm{L}}^{ZZ,(1)}\,\tilde{\Sigma}^{\chi\chi,(1)} = 0$$

  $\approx$ 4000 diagrams

  [*G. W., R. Scharf, M. Böhm '94*]

- SM: many particles, different masses

  ⇒ large number of diagrams:

  1-loop: $\mathcal{O}(10^2)$

  2-loop: $\mathcal{O}(10^3)$

  complicated expressions

- MSSM: duplication of fields compared to SM

  huge number of interaction vertices

  in unconstrained MSSM: 105 new parameters

- QCD, higher-loop calculations:

  e.g. 4-loop β-function

  $\approx$ 50000 diagrams
  [*T. van Ritbergen, J. Vermaseren, S. Larin '97*]

⇒ automation necessary

Examples for program packages

FeynArts, *FeynCalc*, *FormCalc*, *TwoCalc*, *LoopTools*, *s2lse*:
[*DFG Forschergruppe, Karlsruhe*]

FeynArts, *FeynCalc*, *TwoCalc*: *Mathematica*
[*J. Küblbeck, A. Denner, M. Böhm '90*]
[*H. Eck '95*], [*T. Hahn '99*]
[*R. Mertig, M. Böhm, A. Denner '91*]
[*G. W., R. Mertig, R. Scharf, M. Böhm '92*]
[*G. W., R. Scharf, M. Böhm '94*]

FormCalc: *Mathematica, FORM*
[*T. Hahn '98*]

LoopTools: *Mathematica, Fortran*
[*T. Hahn '98*]

s2lse: *C*
[*S. Bauberger, F. Berends, M. Böhm, M. Buza '95*]

GEFICOM, QGRAF, MATAD, MINCER

GEFICOM: *Mathematica, AWK, PERL*
[*K. Chetyrkin, M. Steinhauser*]

QGRAF: *Fortran*
[*P. Nogueira '93*]

MATAD, MINCER: *FORM*
[*M. Steinhauser '96*]
[*S. Gorishny, S. Larin, L. Surguladze, F. Tkachov '89*]
[*S. Larin, F. Tkachov, J. Vermaseren '91*]

DIANA, QGRAF, SHELL2

DIANA: C
[*L. Avdeev, J. Fleischer, M. Kalmykov, M. Tentyukov '97*]

SHELL2:
[*J. Fleischer, O. Tarasov '92*]

xloops, ginac

xloops: *Maple, C*
[*L. Brücher, J. Franzkowski, D. Kreimer '97*]

ginac: C
[*C. Bauer, A. Frink, R. Kreckel '99*]

CompHEP

[*A. Pukhov et al. '89*]

symbolic part: *C*

numerical part: *Fortran, C*

GRACE

[*MINAMI-TATEYA collaboration '87*]

C, Fortran

...

3. Examples of higher-order results in the SM and the MSSM

Theoretical prediction for M_W in the SM:

Definition of Fermi constant G_μ via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta q)$$

Δq: QED corrections in Fermi Model, included in definition

[*R. Behrends, R. Finkelstein, A. Sirlin '56*]

[*T. van Ritbergen, R. Stuart '99*]

Muon decay in SM:

$$\Rightarrow \quad M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1 + \Delta r)\,,$$

$\Updownarrow$

loop corrections

$\Rightarrow$ Theoretical prediction for M_W in terms of M_Z, α, G_μ

SM prediction for Δr, one-loop:

[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \quad \Delta\alpha \quad - \quad \frac{c_W^2}{s_W^2}\Delta\rho \quad + \quad \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Higher-order QCD corrections:

Results in $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha\alpha_s^2)$ known

[*A. Djouadi, C. Verzegnassi '87*], [*A. Djouadi '89*]

[*B.A. Kniehl '90*], [*F. Halzen, B.A. Kniehl '91*]

[*L. Avdeev, J. Fleischer, S.M. Mikhailov, O. Tarasov '94*]

[*K. Chetyrkin, J. Kühn, M. Steinhauser '95*]

EW 2-loop corrections:

Expansion for asymptotically large m_{t}:

Leading $m_{\mathrm{t}}^4/M_{\mathrm{W}}^4$ terms:

[*J. van der Bij, F. Hoogeveen '87*]

[*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*]

[*J. Fleischer, O.V. Tarasov, F. Jegerlehner '93*]

Next-to-leading $m_{\mathrm{t}}^2/M_{\mathrm{W}}^2$ terms:

expansions:

$$M_{\mathrm{W}}, M_{\mathrm{Z}}, M_{\mathrm{H}} \ll m_{\mathrm{t}}$$

$$M_{\mathrm{W}}, M_{\mathrm{Z}} \ll m_{\mathrm{t}}, M_{\mathrm{H}}$$

$+$ interpolation

[*G. Degrassi, P. Gambino, A. Vicini '96*]

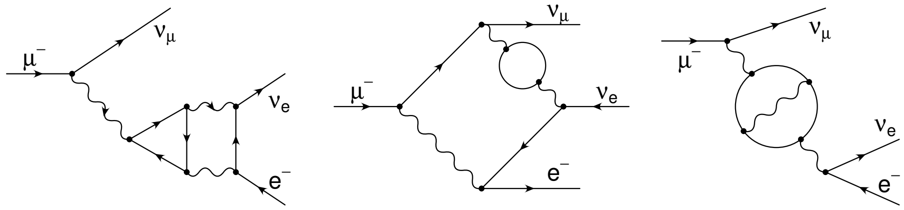
[*G. Degrassi, P. Gambino, A. Sirlin '97*]

$\rightarrow$ Same order of magnitude as $m_{\mathrm{t}}^4/M_{\mathrm{W}}^4$ terms, same sign

$\Rightarrow$ Exact evaluation of EW 2-loop contributions desirable (no expansion in masses)

Complete fermionic two-loop corrections to M_{W}:

Two-loop fermionic contributions to muon decay:



⇒ full dependence on m_{t}, complete light fermion contributions at two-loop order

[*A. Freitas, W. Hollik, W. Walter, G.W. '00*]

External momenta and masses negligible

⇒ Two-loop vacuum diagrams

On-shell renormalization (physical parameters)

⇒ Two-loop 2-point functions, $p \neq 0$

Method of calculation:

Algebraic evaluation:

FeynArts [*J. Küblbeck, A. Denner, M. Böhm '90*]
[*H. Eck '95*], [*T. Hahn '99*]

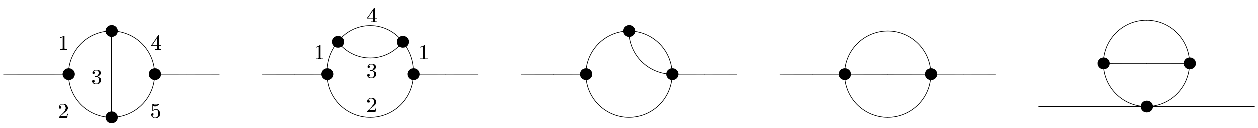
Generation of Feynman amplitudes, counterterms

TwoCalc [*G. W. '92*], [*G. W., R. Scharf, M. Böhm '94*]

Algebraic evaluation, tensor integral reduction, etc.

⇒ General two-loop 2-point functions reduced to standard scalar integrals

$$T_{i_1 i_2 \ldots i_l} = \int \frac{d^D q_1}{i\pi^2 (2\pi\mu)^{D-4}} \int \frac{d^D q_2}{i\pi^2 (2\pi\mu)^{D-4}} \times \frac{1}{\left[k_{i_1}^2 - m_1^2\right]\left[k_{i_2}^2 - m_2^2\right] \cdots \left[k_{i_l}^2 - m_l^2\right]},$$



Calculation in general R_ξ gauge: ξ_A, ξ_Z, ξ_W

⇒ Check of Ward identities, gauge independence

Regularisation:

SM: DREG [*G. 't Hooft, M. Veltman '72*]

MSSM: DRED [*W. Siegel '79*], [*D. Capper et al. '80*]

Careful treatment of γ_5 necessary

Two-loop renormalization:

On-shell scheme (physical parameters)

Numerical evaluation:

s2lse [*S. Bauberger, F. Berends, M. Böhm, M. Buza '95*]
[*S. Bauberger, M. Böhm '95*]

One-dimensional integral representations with elementary functions

$\Rightarrow$ Exact evaluation of 2-loop 2-point functions

Results:

Checks of the calculation:

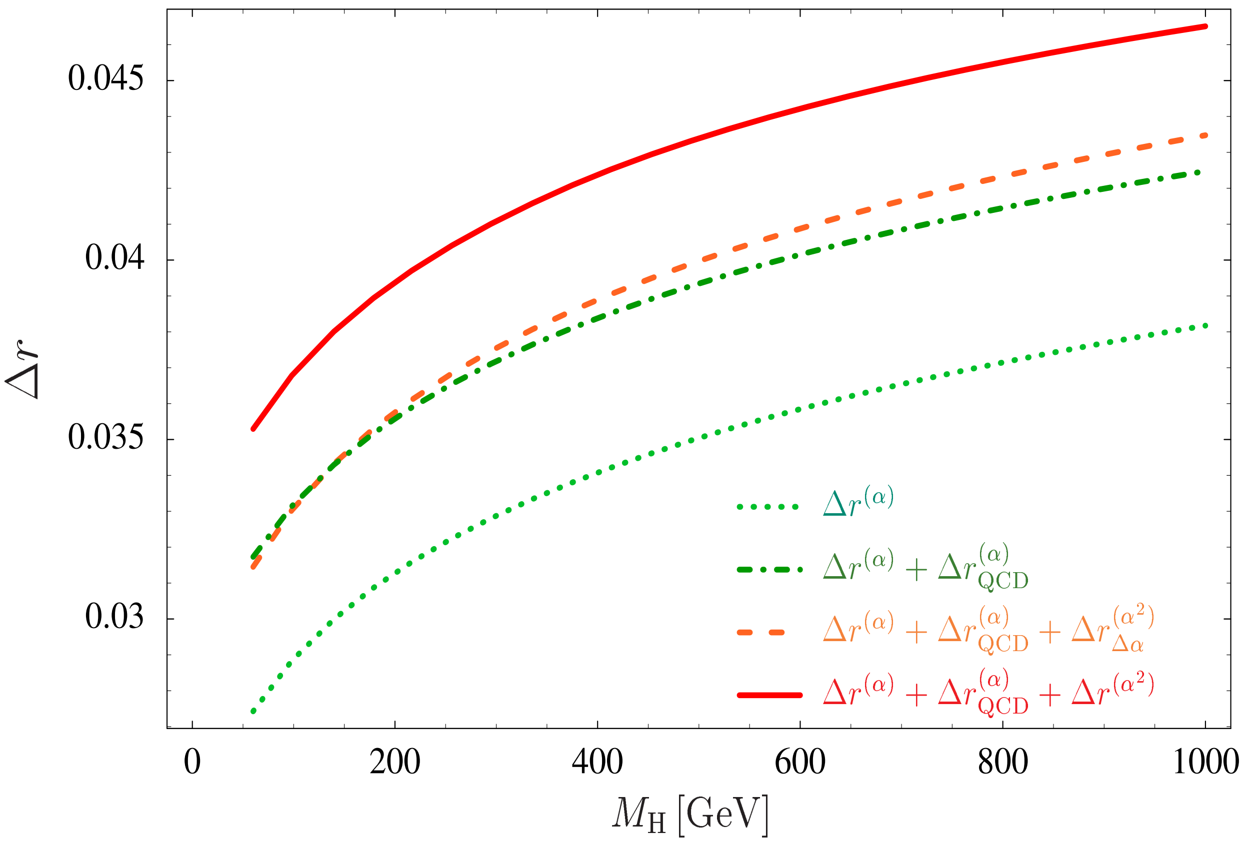
UV-, IR-finiteness,
general R_ξ-gauge $\Rightarrow$ gauge-parameter independence at algebraic level

Higgs-mass dependence agrees with existing result
[*S. Bauberger, G.W. '98*]

1-loop, QCD and ew 2-loop contributions to Δr:

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}$$

fully expanded, no resummation needed



Comparison of complete fermionic two-loop result with expansion in m_t:

M_H/ GeV	M_W/ GeV	M_W^{expan}/ GeV	ΔM_W/ MeV
65	80.3997	80.4039	-4.2
100	80.3771	80.3805	-3.4
300	80.3051	80.3061	-1.0
600	80.2521	80.2521	0.0
1000	80.2134	80.2129	0.5

$\Rightarrow$ Agreement within $\approx$ 4 MeV

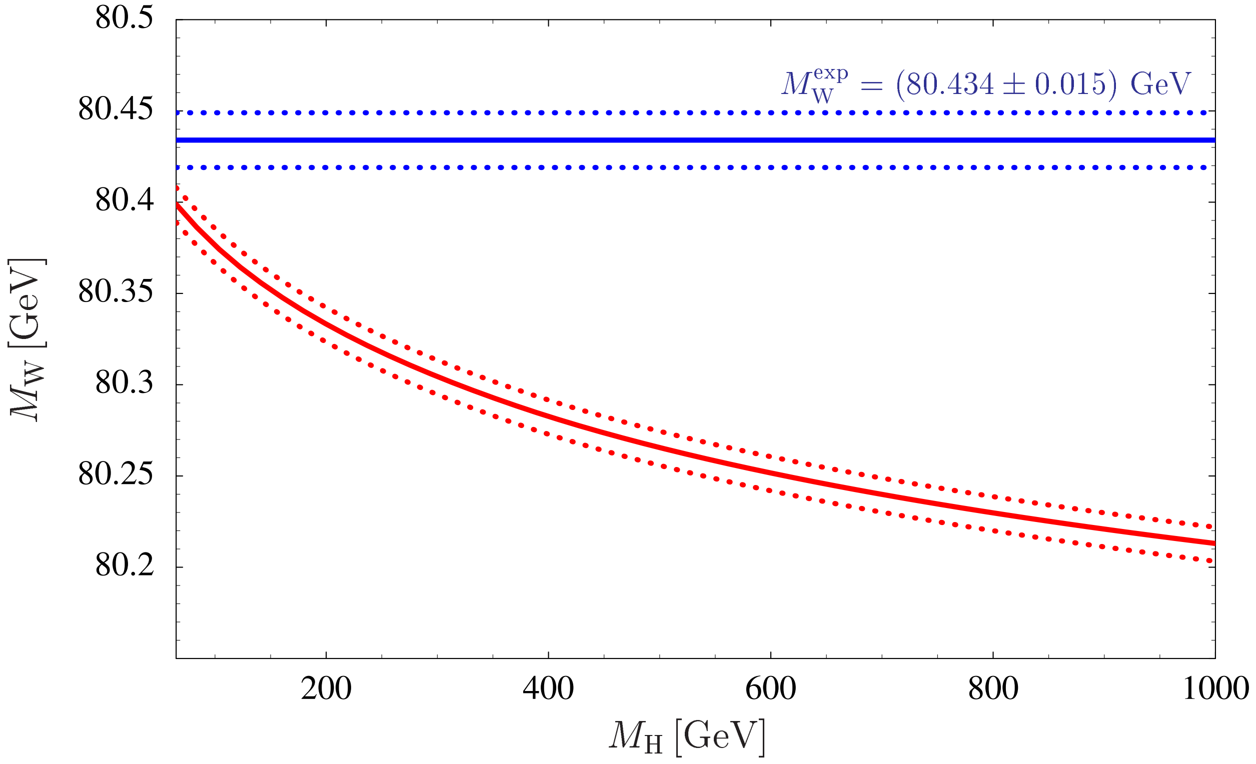
Estimate of theoretical uncertainty from unknown higher-order corrections:

- Purely bosonic $\mathcal{O}(\alpha^2)$ electroweak corrections
- Higher-order electroweak corrections: $\mathcal{O}(\alpha^3)$
- Higher-order QCD corrections: $\mathcal{O}(\alpha\alpha_s^3)$, $\mathcal{O}(\alpha^2\alpha_s)$

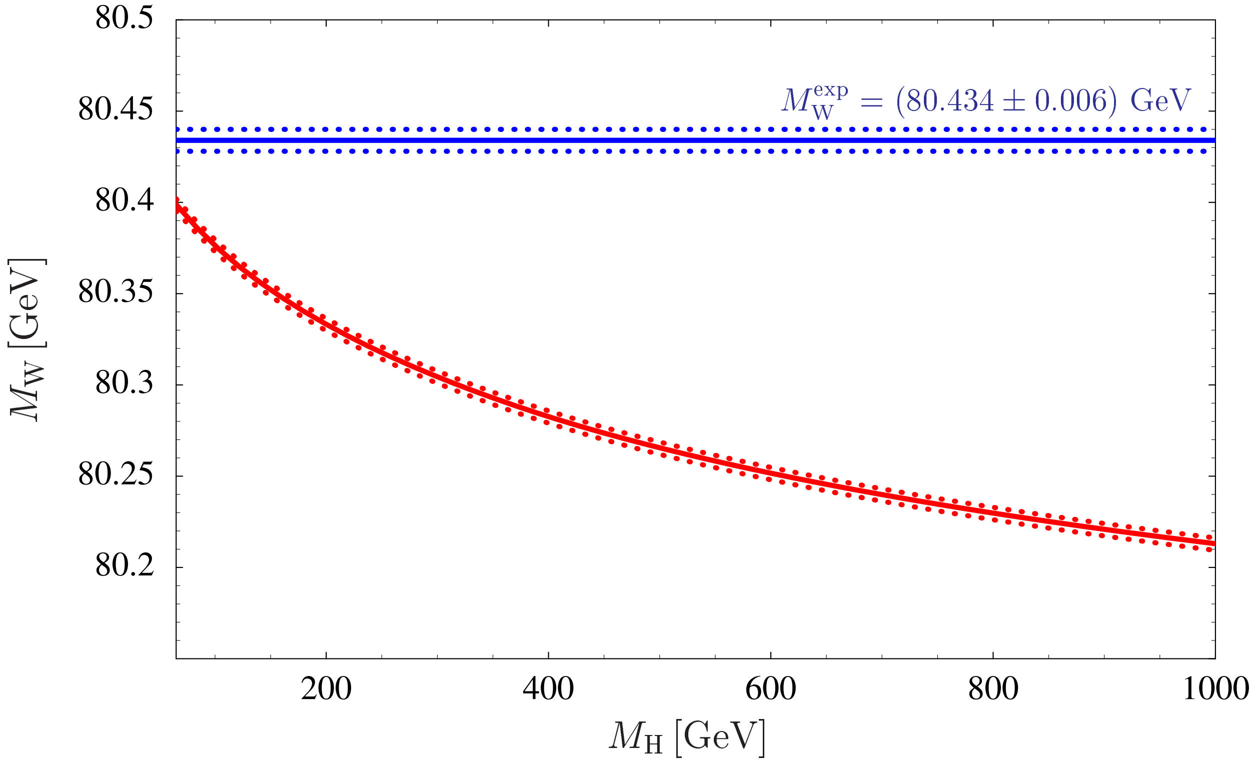
$$\Rightarrow \quad \Delta M_W^{theo} \lesssim 6\,\text{MeV}$$

$M_{\mathsf{W}}(M_{\mathsf{H}})$ in the SM, prospective future accuracies:

LHC precision:



GigaZ precision:



Applications of *FeynArts*, *TwoCalc* in the MSSM:

SUSY model file implemented into *FeynArts*
[*T. Hahn '99*]

<u>Precision observables in the MSSM:</u>

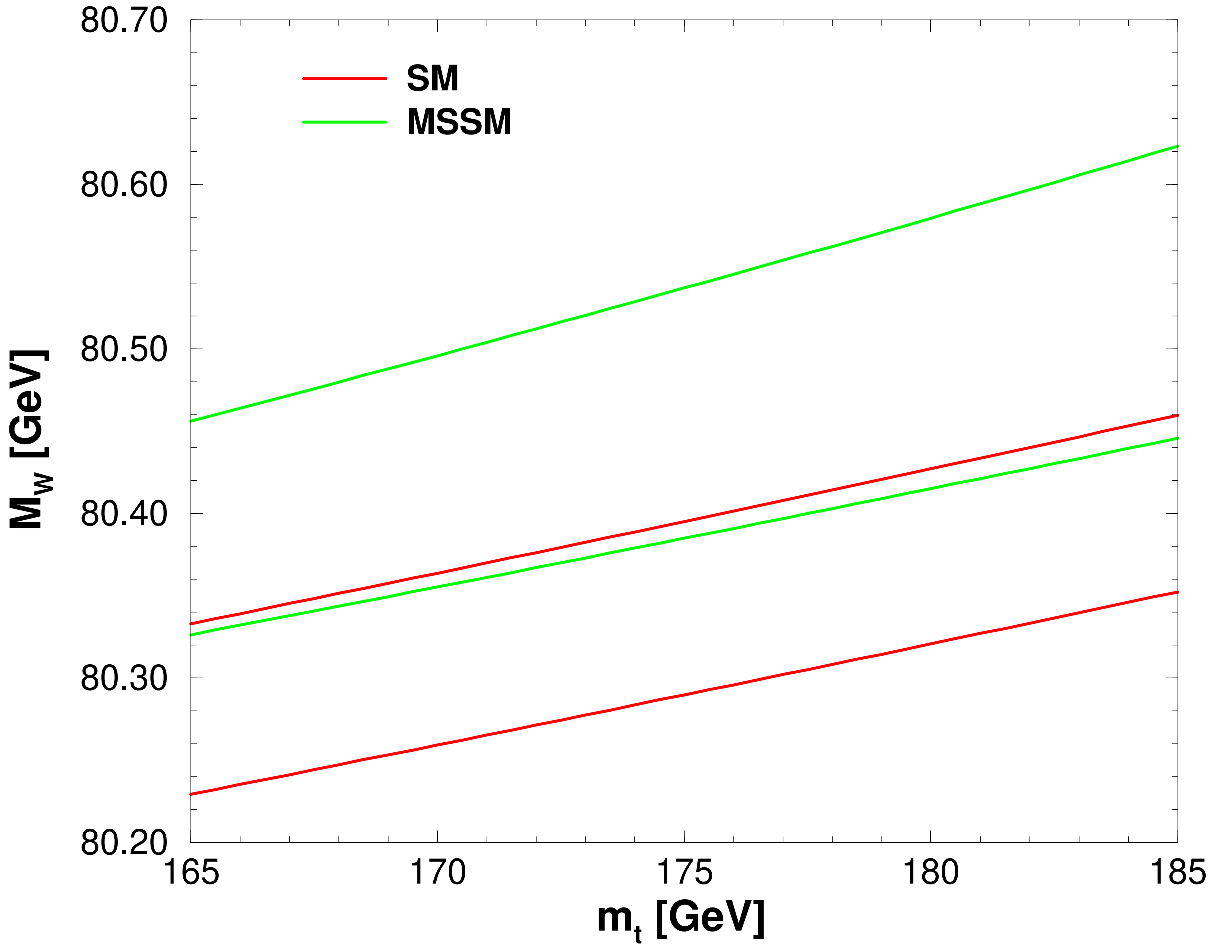
In order to treat MSSM at same level of accuracy as SM
$\Rightarrow$ Include higher-order corrections

Prediction for M_{W}, $\sin^2\theta_{\text{eff}}$ in the MSSM:

- Complete 1-loop results
  [*P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94*]
  [*D. Garcia, J. Solà '94*]
  [*D. Garcia, R. Jiménez, J. Solà '95*]
  [*A. Dabelstein, W. Hollik, W. Mösle '95*]
  [*P. Chankowski, S. Pokorski '96*]

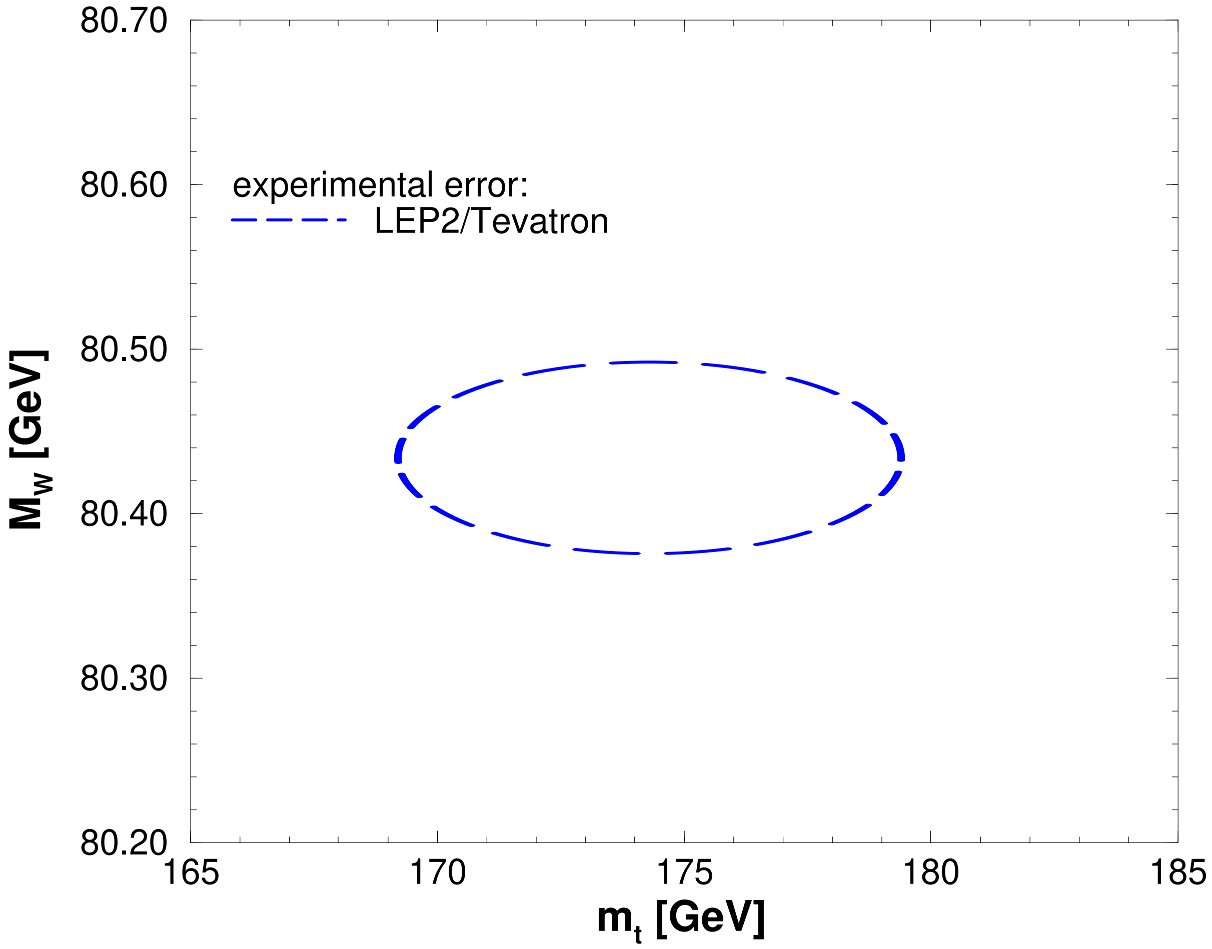
- Leading 2-loop $\mathcal{O}(\alpha\alpha_{\text{s}})$ corrections
  [*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]
  [*S. Heinemeyer, W. Hollik, G. W. '98*]

Prediction for M_W in the SM and the MSSM:

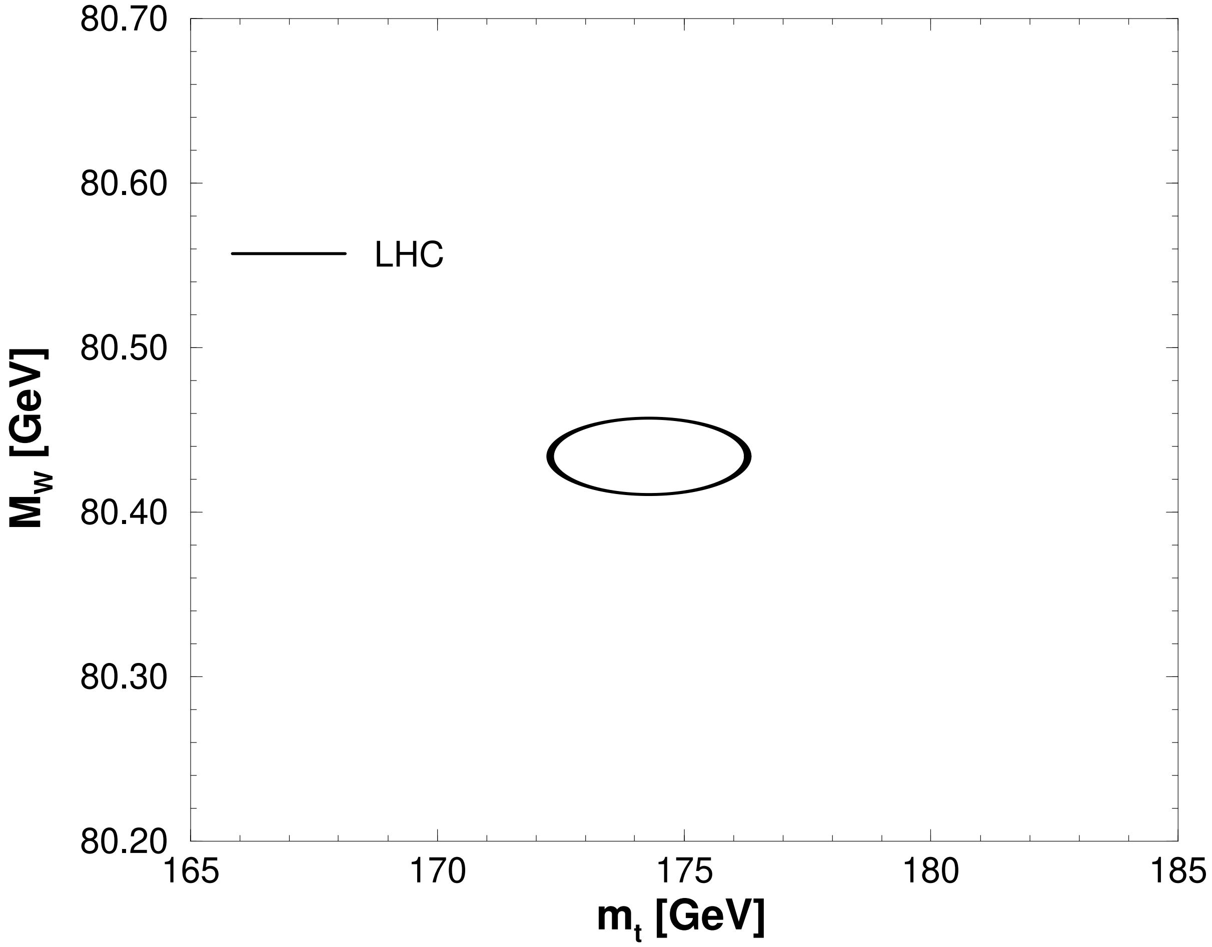


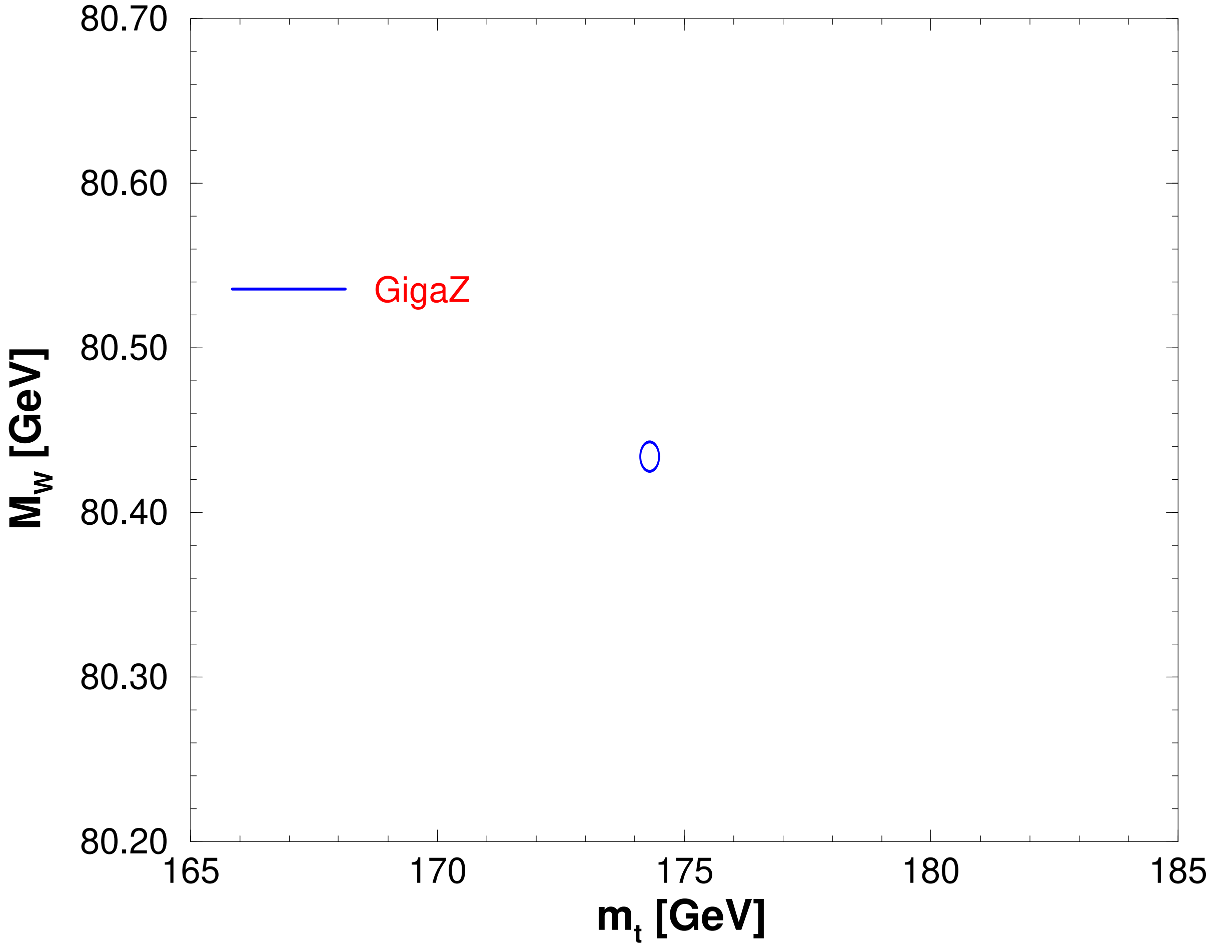
SM: M_H varied

MSSM: SUSY parameters varied



80.70
80.60
80.50
80.40
80.30
80.20
165
170
175
180
185
experimental error:
LEP2/Tevatron
M_W [GeV]
m_t [GeV]


80.70
80.60
80.50
80.40
80.30
80.20
LHC
M_W [GeV]
165
170
175
180
185
m_t [GeV]



80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
GigaZ
165
170
175
180
185
m_t [GeV]

Stringent direct test of SUSY:

Light Higgs boson h required

m_{h} calculable from MSSM parameters:

Tree level: $m_{\mathrm{h}} < M_{\mathrm{Z}}$

Large radiative corrections: $\Rightarrow$ $m_{\mathrm{h}} \lesssim 135$ GeV

m_{h}, $\sigma_{e^+e^-\to Zh}$, Γ_{h}:

Precise prediction needed for Higgs search:
Discovery/exclusion potential of LEP2 and upgraded Tevatron

High-precision measurement of m_{h}:

$$\begin{aligned} \text{LHC}: &\quad \Delta m_{\mathrm{h}} \approx 0.2 \text{ GeV} \\ \text{LC}: &\quad \Delta m_{\mathrm{h}} \approx 0.05 \text{ GeV} \\ \text{Muon Collider}: &\quad \Delta m_{\mathrm{h}} \approx 0.1 \text{ MeV} \end{aligned}$$

$\Rightarrow$ Very sensitive test of the theory

Two-loop results for m_{h} in the MSSM

Higgs sector of the MSSM:

Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$\begin{aligned} V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ &\quad + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \end{aligned}$$

Physical states: $h^0, H^0, A^0, H^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_{\mathrm{A}}^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

Diagonalization of tree-level mass matrix

$\Rightarrow m_{\mathrm{h}}, m_{\mathrm{H}}$, mixing angle α

Tree-level result for m_{h}, m_{H}:

$$m^2_{\mathrm{H,h}} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow$ $m_{\mathrm{h}} \leq M_Z$ at tree level

Large radiative corrections:

Yukawa couplings: $\frac{e\,m_t}{2M_W s_W}$, $\frac{e\,m_t^2}{M_W s_W}$, ...

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

Previously used for LEP analyses:

RG improved one-loop effective potential result

$\Rightarrow$ Leading logs at two-loop level

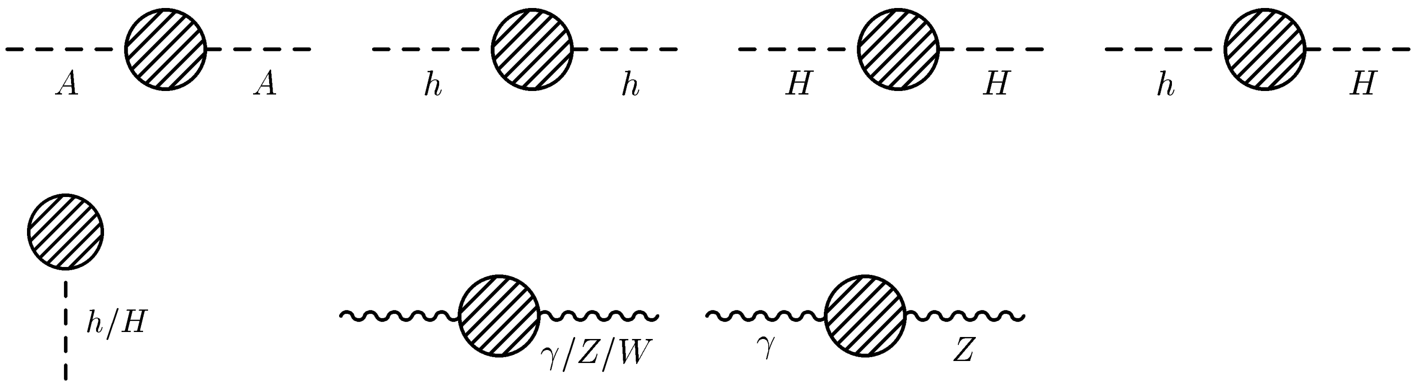
program: *subhpole*

[*M. Carena, M. Quirós, C. Wagner '95*]

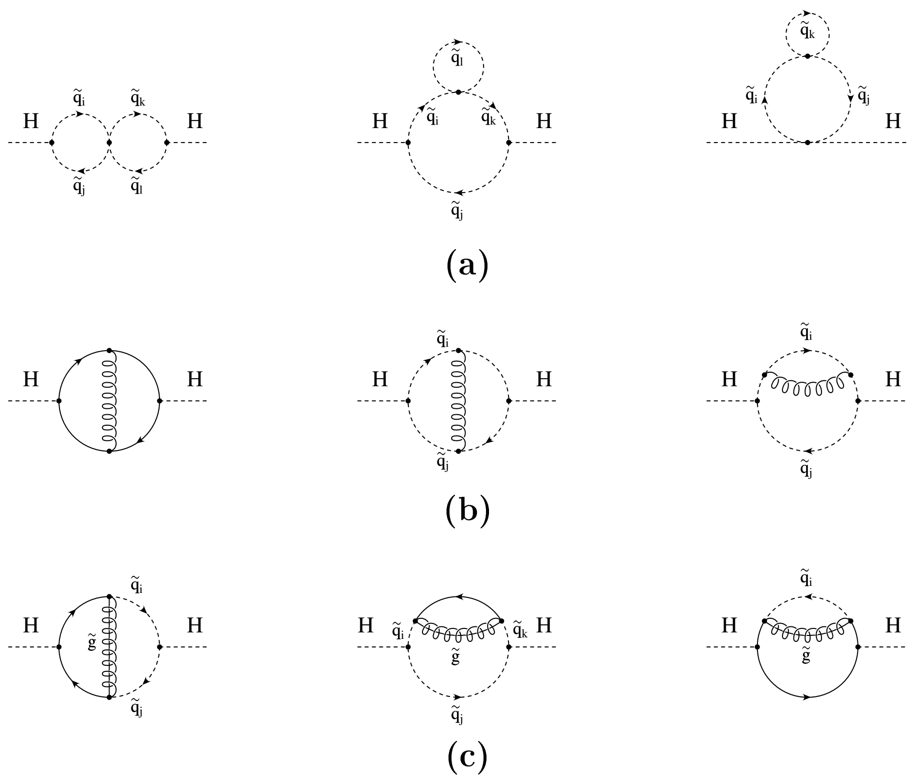
New result:

Feynman-diagrammatic two-loop result

Explicit Feynman-diagrammatic calculation,
Higgs mass: pole of propagator



Complete one-loop + dominant two-loop corrections



Fortran codes: *FeynHiggs*, *FeynHiggsFast*
[*S. Heinemeyer, W. Hollik, G. W. '98, '00*]

`http://www.feynhiggs.de`

Upper bound on m_{h} in the MSSM:

Unconstrained MSSM:

M_{A}, $\tan\beta$, 5 parameters in $\tilde{\mathrm{t}}$–$\tilde{\mathrm{b}}$ sector, μ, $m_{\tilde{\mathrm{g}}}$, M_2

$$\Rightarrow \; m_{\mathrm{h}} \lesssim 130\,\mathrm{GeV}$$

for $m_{\mathrm{t}} = 175\,\mathrm{GeV}$, no theoretical uncertainties
[*S. Heinemeyer, W. Hollik, G. W. '99*]

Holds also for $\mathcal{CP}$-violating case
[*A. Pilaftsis, C. Wagner '99*]

Theoretical uncertainties:

– From unknown higher-order corrections

$\Delta m_{\mathrm{h}} \approx \pm 3$ GeV

– From uncertainties in input parameters

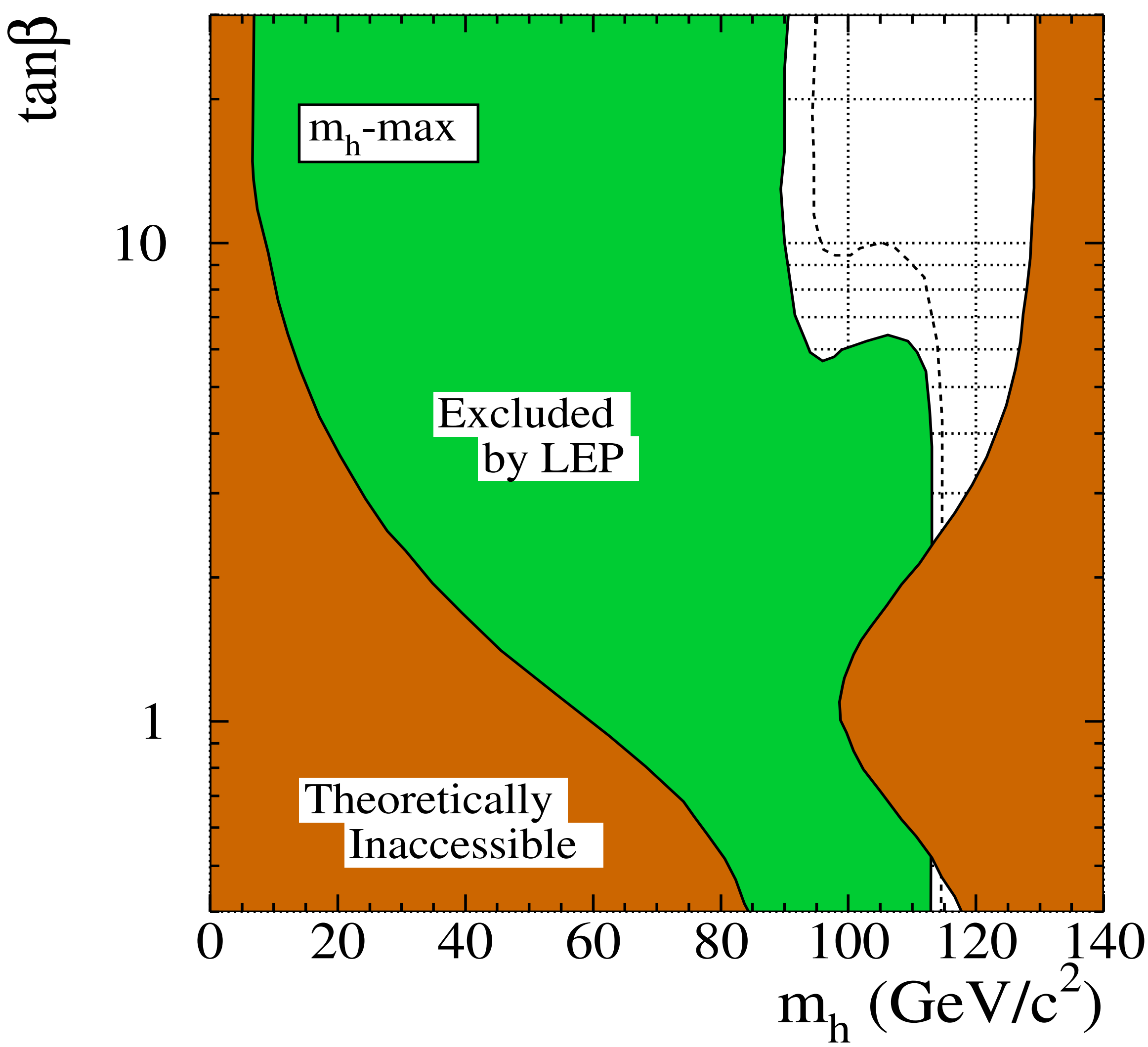
m_{t}, ... , M_{A}, $\tan\beta$, $m_{\tilde{\mathrm{t}}_1}$, $m_{\tilde{\mathrm{t}}_2}$, $\theta_{\tilde{\mathrm{t}}}$, $m_{\tilde{\mathrm{g}}}$, ...

$\Delta m_{\mathrm{t}} = 1$ GeV $\Rightarrow$ $\Delta m_{\mathrm{h}} = 1$ GeV

Experimental search vs. upper m_h-bound (*FeynHiggs*)

[*LEP Higgs Working Group '00*]

m_h^{max}-scenario:



$$m_h > 89.9\,\text{GeV (expected: 93.8 GeV)}, \quad 95\% \text{ C.L.}$$

$$M_A > 90.5\,\text{GeV (expected: 94.1 GeV)}$$

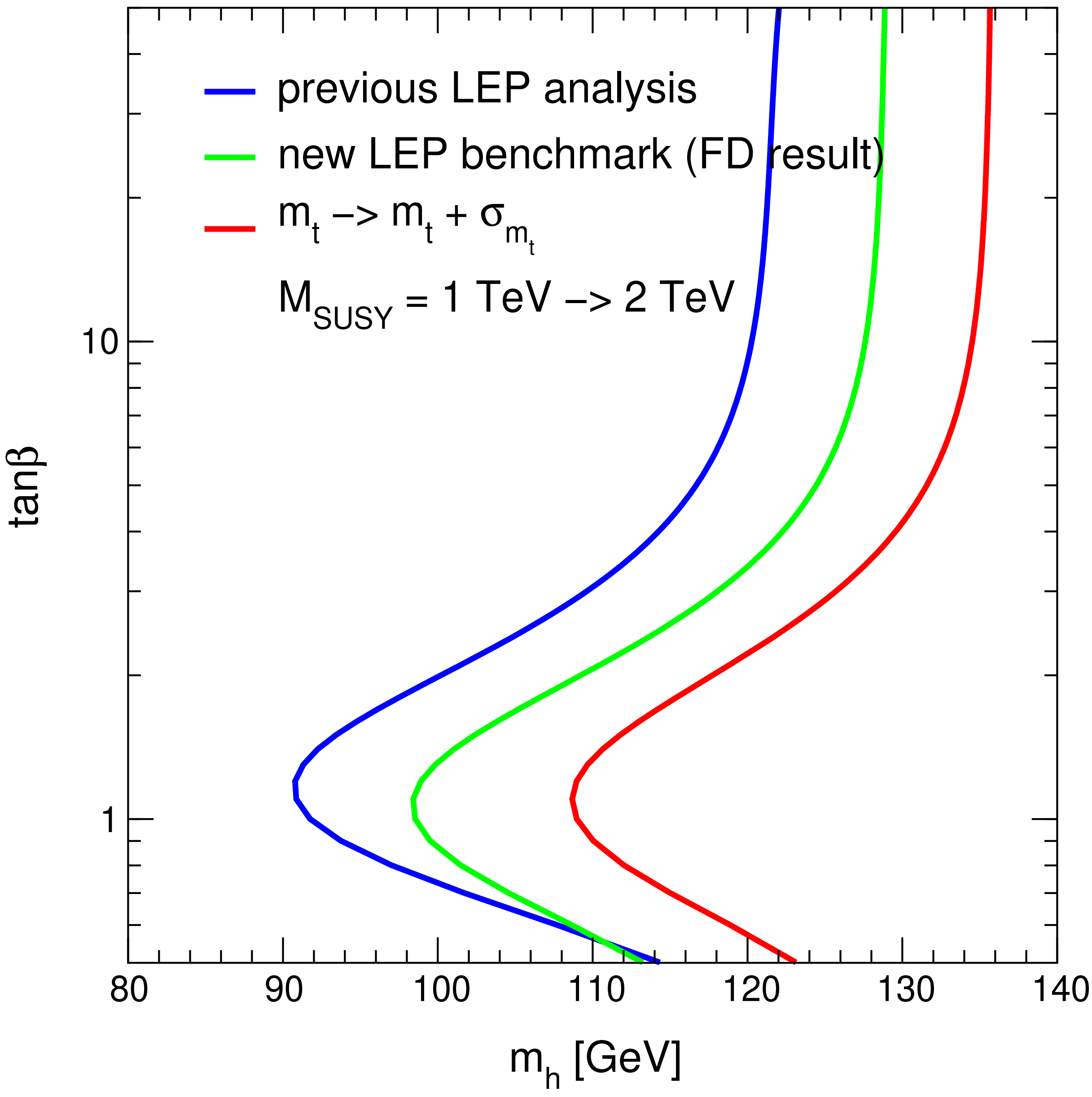
Excluded $\tan\beta$ region:

$$0.5 < \tan\beta < 2.3$$

for $m_t = 174.3$ GeV, $M_{SUSY} = 1$ TeV

Effect of: $m_t \to m_t + \sigma_{m_t}$, $M_{SUSY} = 1$ TeV $\to$ 2 TeV

[*S. Heinemeyer, W. Hollik, G. W. '00*]



$\Rightarrow$ $\tan\beta$ exclusion only for $m_h^{exp} \gtrsim 110$ GeV

$\Rightarrow$ precise knowledge of m_t important!

4. Application of computer-algebra systems beyond tools for calculations

Computer algebra information sources, networks:

ISSAC, CAIN, SymbolicNet, ...

In Germany: Fachgruppe Computeralgebra

Physics representative: `Georg.Weiglein@cern.ch`

Computer algebra benchmarks:

SymbolicData Project, `www.symbolicdata.org`

Computer algebra for scientific publishing:

$\Rightarrow$ errors from typing formulas avoided

results directly accessible

$\Rightarrow$ worldwide standard necessary

Current standard in high-energy physics:

– language: English

– text processing: $\LaTeX$

– electronic preprint server/archive: `xxx.lanl.gov`

worldwide accessible, free of charge

Increased need for communication:

different CA systems, numerical packages,
data bases, expert systems, text processing, ...

$\Rightarrow$ general problem-solving environments

$\Rightarrow$ standardization:

integration of program parts

data transfer between systems:
transport mechanism, data representation

$\Rightarrow$ OpenMath initiative

`www.nag.co.uk/projects/openmath/omsoc`

Internet-related applications:

– use of network computing facilities: *JAVA*, ...

– CA for cryptographic techniques

...

5. Conclusions

- High-energy physics triggered development of computer algebra

  CA in extensive use for several decades

- Applications for higher-order corrections in the electroweak theory: SM and MSSM

  Many of todays calculations not feasible without CA

- Future improvements of soft- and hardware:

  $\Rightarrow$ extension of applicability

  highly sophisticated computations

  general problem-solving environments