Simple Scaling for Faster Tracking Simulation in Accelerator Multiparticle Dynamics

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Abstract. Macroparticle tracking is a direct and attractive approach to following the evolution of a phase space distribution. When the particles interact through short range wake fields or when inter-particle force is included, calculations of this kind require a large number of macroparticles. It is possible to reduce both the number of macroparticles required and the number of tracking steps per unit simulated time by employing a simple scaling which can be inferred directly from the single-particle equations of motion. In many cases of practical importance the speed of calculation improves with the fourth power of the scaling constant. Scaling has been implemented in an existing longitudinal tracking code; early experience supports the concept and promises major time savings. Limitations on the scaling are discussed.

INTRODUCTION

Multiparticle tracking programs have a long history and established utility for modeling the evolution of the longitudinal phase space distributions for particles in accelerators as the particles respond to the rf in acceleration or bunch manipulation. The macroparticle distribution can be used to approximate the evolution of the beam current distribution or fourier spectrum throughout the process being modeled. Collective behavior of the beam is modeled by calculating the beam current every time step and applying the forces arising from it to the single particle motion. Such calculations raise questions about the number of macroparticles needed and the relevant bandwidth for quantities calculated in frequency domain which require careful attention. It is very easy to generate spectacular spurious instabilities by an insufficient number of macroparticles. Recent studies of high brightness injectors and the so-called factory accelerators have renewed interest in these questions.

SCALING CONCEPT

The objective of the scaling is to allow more rapid calculation of the time evolution of the energy and rf phase of particles in a synchrotron or storage ring. A more complete exposition of the principles has been published.[1] What follows is an effort to stimulate interest and possible further developments. Gerasimov[2] has developed scaling rules for Fokker Planck simulations of beam cooling which are likewise used to accelerate macroparticle modeling. Perhaps simillar tactics will work elsewhere.

A suitable single turn map for the macroparticles is[3]

$$\varphi_{i,m} = \varphi_{i,m-1} + \frac{2\pi h\eta}{\beta_s^2 E_s}\varepsilon_{i,m-1} \quad (1)$$

$$\varepsilon_{i,m} = \varepsilon_{i,m-1} + eV(\varphi_{i,m} + \phi_{s,m}) - eV(\phi_{s,m}) \ , \quad (2)$$

where φ is a phase difference between a particle and the synchronous phase ϕ_s, likewise ε is the energy difference between a particle and the synchronous energy E_s, i labels particles, and m labels turns.

By inspection of the map parameters it is plausible that the phase space motion can be accelerated by scaling the phase slip factor η and the potential by the same factor. The potential is not necessarily just that from the rf system; the collective potential produced by the beam current enters identically.

There is a reduction by a factor λ^{-1} in the computing time by speeding up the clock in the scaled calculation. However, scaling up the time means that frequencies associated with the motion, like the rf frequency for example, are also scaled up. A consequence of the frequency scaling is that a resonant potential in the problem, like a higher order cavity mode for example, must be entered into the calculation with its resonant frequency scaled up by the same factor. If this is done, the result of scaled and original mappings are practically indistinguishable from

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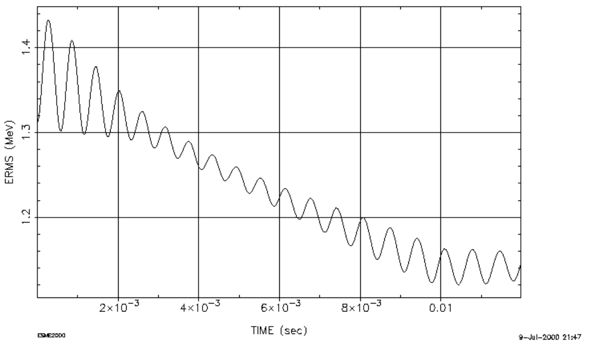

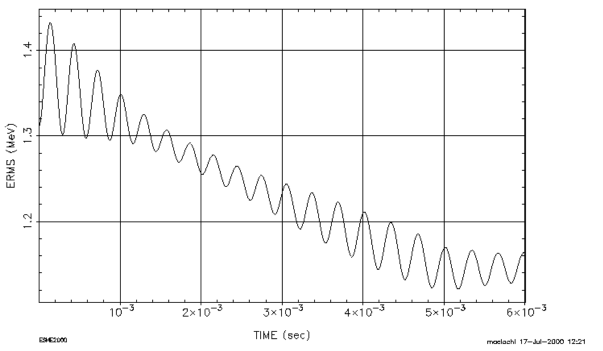

FIGURE 1. (left) The rms energy spread *vs.* time in the ring specified in Table II for a beam intensity of $2 \cdot 10^{13}$ protons including both an h=3 HOM and the perfectly conducting wall space charge force **FIGURE 2.** (right) The same conditions used for Fig. 1 except scaled with $\lambda = 2$ Note that the time scale is $t' = t/\lambda$, one half of that in Fig. 1.

one another for a reasonable choice of λ when the comparison is made at a scaled time $t' = t/\lambda$.

When broadband impedance or the direct interparticle forces are included, the consequences of the frequency scaling are different and very advantageous. When the effects of beam pipe impedance must be covered over some particular frequency bandwidth, or, more or less equivalently wake fields down to some minimum range, the charge distribution must be divided into bins sufficiently narrow to represent detail in the distribution to that scale. The bandwidth of interest must be spanned by fourier components of the beam current. If beam circulation frequency is $f_\circ$ and the required bandwidth is W, the charge distribution must be divided into $2W/f_\circ$ bins for the finite transform. However, in the scaled system $f_\circ$ is λ times higher; the number of bins can be reduced by a factor λ^{-1}. The number of harmonics is reduced, but the frequency range covered is the same. What has been sacrificed is sensitivity to features in the frequency dependence of the longitudinal impedance $Z_{\|}$ on the scale of $f_\circ$. However for space charge there is no sacrifice at all. In frequency domain the equivalent impedance is

$$\frac{Z_{sc}}{n} = -i\frac{Z_\circ g}{2\beta\gamma^2} \;, \qquad (3)$$

where n is the harmonic number, $Z_\circ = \sqrt{\mu_\circ/\varepsilon_\circ} = 377\Omega$, $\gamma = E/m_\circ c^2$, and g is the geometric factor for the beam tube. The evaluation of this term is not compromised by more widely spaced frequency sampling. For wideband impedances is general, there will be little loss in information for reasonable choices of λ.

What has been gained is the opportunity to reduce the the number of macroparticles required for a given level of numerical noise in the mapping. It has been shown that when a three point difference formula for the derivative of the linear charge density is used in calculating the space charge force in time domain, the number of macroparticles can be reduced by a factor λ^{-3} when the number of bins is reduced by the factor λ^{-1}.[4] An argument based on sampling in the frequency domain shows that more generally this result applies to any source of longitudinal impedance with a frequency dependence featureless on the scale of $\Delta f \sim$ few $\times f_\circ$.[5] Thus, the big payoff comes not by speeding up the clock but by reducing the number of particles. However, if the important impedance is a narrow resonance, the macroparticle count can not be reduced by scaling, leaving only the first power effect. Nonetheless, for a broad class of applications the gain from scaling will be a factor λ from the reduced number of iterations and an additional factor of λ^3 from the reduced macroparticle count.

NUMERICAL COMPARISON

The effectiveness of the scaling is illustrated by comparing an un-scaled and a scaled calculation of the evolution of the energy spread in a storage ring. The parameters, simillar to those of the Los Alamos Proton Storage Ring, are given in the Table. The collective potential is generated by a hypothetical spurious cavity mode at h=3 and the perfectly conducting wall force from $2 \cdot 10^{13}$ protons. To evaluate the space charge force, the gradient of the charge distribution is calculated; there must be enough macroparticles to produce a smooth distribution. An insufficient number results in large local fluctuation of the force and spurious breakup of the beam into small clumps. In Figs. 1 and 2 the evolution of the rms energy spread is shown as the charge is raised from zero to a final value of $2 \cdot 10^{13}$ protons over .012 seconds. The un-scaled example in Fig. 1 needed $2 \cdot 10^6$ macroparticles and took 32 hours of processor time on a Sun Ultra 2. The plot in

Fig. 2 was generated from tracking with the scaling $\lambda = 2$ and used $2.5 \cdot 10^5$ macroparticles; it required 1.9 hours on the same computer, just a little better than λ^4 times faster. The choice of two for λ was arbitrary; too large a value can give incorrect results.

UTILITY

If the scaling factor λ can be significantly larger than one without reducing the accuracy of the result, much time can be saved in macroparticle modeling. One limitation on the choice of λ has been noted, *viz.*, that the spacing of harmonics of the circulation angular frequency $\omega_\circ$ should not obscure important features of the frequency dependence of $Z_{\|}(\omega)$. Returning to the difference equations eqs. 1 and 2, one sees that scaling by integral λ is equivalent to a multi-turn map. This indicates that an additional feature of the scaling with $\lambda > 1$ is the introduction of an artificial increase in the effective time step. This artificial discretization error leads to inaccurate macroparticle trajectories that can be readily apparent in extreme cases. The parameter to control is the synchrotron oscillation phase advance between energy increments. When this is below 0.01π or so, the calculation should be at least qualitatively reasonable. Another condition, which is generally the same as the limitation on synchrotron oscillation phase advance per iteration is that rf phase slip per iteration should be small with respect to a bunch length, and energy increment per iteration should be small with respect to the beam energy spread. The scaling described is so simple to implement and so innocuous in typical applications that it seems reasonable to employ it when calculation time for the un-scaled case is more than a few minutes. The precaution of considering the granularity of the sampling of $Z_{\|}(\omega)$ requires forethought, but the test for the appropriateness of the iteration step and the adjustment of the frequency of any narrow resonances can easily be handled by the modeling code.

The discussion has used mostly a frequency domain description for simplicity. The example, however, is a pure time domain calculation using a slightly modified version of the ESME tracking code.[6] A version now available, called ESME2001, incorporates λ scaling as an option; it appears compatible with all other features in both frequency and time domain. The scaling of potentials, frequency, and time are automatic, but no tests for the appropriateness of the scaling constant have been included.

Table 1. Storage ring parameters used in tracking examples

Parameter	Symbol	Value	Units
Circumference	C	90.261	m
momentum	p	1459.7	MeV/c
transition energy/$m_\circ c^2$	γ_T	3.08	
slip factor $\gamma_T^{-2} - \gamma^{-2}$	η	.18694	
rf peak voltage	$V_{\rm rf}$	5	kV
rf harmonic	h	1	
circulation frequency	$f_\circ$	2.7940	MHz
HOM cavity mode			
frequency ($h \approx 3$)	$f_{\rm res}$	8.4658	MHz
resonant impedance	$R_{\rm sh}$	300	Ohm
quality factor	Q	100	

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6. The current documentation, containing references to the underlying principles, is most accessible on the ESME web page `www-ap.fnal.gov/ESME`