Two-loop Calculations in the MSSM with FeynArts

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Abstract. Recent electroweak two-loop corrections in the Minimal Supersymmetric Standard Model (MSSM) are reviewed. They have been obtained with the help of the programs *FeynArts* and *TwoCalc*, making use of the recently finished MSSM model file for *FeynArts*. Short examples of how to use the two codes together with the analytic result for the $O(G_F^2 m_t^4)$ corrections to the ρ-parameter in the MSSM are presented.

INTRODUCTION

Theories based on Supersymmetry (SUSY) are widely considered as the theoretically most appealing extension of the Standard Model (SM). The Minimal Supersymmetric Standard Model (MSSM) predicts the existence of scalar partners $\tilde{f}_L, \tilde{f}_R$ to each SM chiral fermion, and spin–1/2 partners to the gauge bosons and to the scalar Higgs bosons, where two Higgs doublets are present in the MSSM. So far, the direct search of SUSY particles at present colliders has not been successful. One can only set lower bounds of $O(100\,\mathrm{GeV})$ on their masses, see Ref. (1). An alternative way to probe SUSY is to search for the virtual effects of the additional particles. The experimental precision has to be matched with high precision theoretical predictions for the various precision observables. The most prominent role in this respect is played by the ρ-parameter, see Ref. (2):

$$\rho = \frac{1}{1-\Delta\rho}, \quad \Delta\rho = \frac{\Sigma_Z}{M_Z^2} - \frac{\Sigma_W}{M_W^2}. \tag{1}$$

The radiative corrections to the vector boson self-energies, $\Sigma_{Z,W}$ constitute the leading, process independent corrections to many electroweak precision observables, such as the W boson mass, M_W, where

$$\delta M_W \approx M_W/2 \;\; c_W^2/(c_W^2 - s_W^2)\, \Delta\rho, \tag{2}$$

with $c_W^2 = 1 - s_W^2 = M_W^2/M_Z^2$. Within the MSSM the corrections have so far been restricted to $O(\alpha\alpha_s)$, see Ref. (3). In order to match the accuracy obtained in the SM and the (prospective) experimental uncertainties, the leading two-loop corrections to $\Delta\rho$ at $O(G_F^2 m_t^4)$ are desirable. These corrections are the lowest order contributions involving non-SM particles with a m_t^4 dependence. In the limit of a large SUSY scale, $M_{SUSY} \gg M_Z$, the SUSY particles decouple from the observables, leaving the two Higgs doublets of the MSSM active. First results for this contribution have recently been calculated in Ref. (4).

Feynman-diagrammatic two-loop calculations involve a large number of diagrams. In the MSSM the additional problem of a proliferation of scales is apparent. Therefore, in order to perform the calculation as presented in Ref. (4), the use of computer algebra programs is inevitable. In particular, we made use of the amplitude generator *FeynArts*, see Ref. (5), where the MSSM model file has recently been completed. The reduction of the amplitudes to scalar integrals have been performed with the program *TwoCalc*, see Ref. (6).

TECHNIQUES FOR TWO-LOOP CALCULATIONS

In order to calculate the $O(G_F^2 m_t^4)$ corrections to the ρ-parameter, the diagrams in Fig. 1 have to be evaluated.

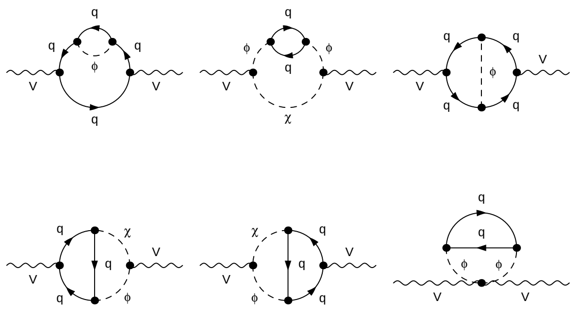

FIGURE 1. Generic diagrams for $O(G_F^2 m_t^4)$ corrections to $\Delta\rho$. ($V = Z, W$, $f = t, b$, $\phi, \chi = h, H, A, H^{\pm}, G, G^{\pm}$)

The diagrams and the corresponding amplitudes have been generated with the help of the *Mathematica* program *FeynArts* and the recently accomplished MSSM model file. This model file contains all relevant information about the MSSM particles and vertices. (Only counter term vertices are missing at present.) *FeynArts* has been checked in several ways to insure its reliability. Apart from many self-energy calculations, whole processes like $e^+e^- \to t\bar{t}$, $e^+e^- \to H^+H^-$, $q\bar{q} \to t\bar{t}$ and $e^+e^- \to \tilde{\chi}^{0,+}\tilde{\chi}^{0,-}$ have been calculated by hand and with *FeynArts*. Perfect agreement has been found for all processes, see Refs. (5, 7).

How easy the evaluation of the amplitudes has become with *FeynArts* is demonstrated in the following sequence from a *Mathematica* session where the Z boson self-energy amplitude, corresponding to Fig. 1, has is obtained. (A detailed guide can be found in Ref. (5).)

(start of *Mathematica* session)

```
> <<FeynArts.m;
```
(→ loading *FeynArts* into *Mathematica*)

```
> se2 = CreateTopologies[2, 1->1,
        ExcludeTopologies->Internal];
```
(→ two-loop topologies are created)

```
> V2V2 = InsertFields[se2,
         V[2] -> V[2],
         Model->"MSSM",
         ExcludeParticles -> {...} ];
```
(→ fields are inserted into the topologies, incoming field `V[2]` = Z and outgoing field `V[2]` are specified, the model is chosen to be the MSSM)

```
> Paint[V2V2];
```
(→ the diagrams are painted (optional), see Fig. 1)

```
> V2V2A = CreateFeynAmp[V2V2];
```
(→ Feynman diagrams are converted into amplitudes)

```
> V2V2A >> V2V2.amp;
```
(→ result for the two-loop amplitude of the Z self-energy is saved in the file `V2V2.amp`)
(end of *Mathematica* session)

The further evaluation of the amplitudes has been performed with the *Mathematica* program *TwoCalc*, see Ref. (6). It performs the reduction of the self-energy amplitude at the two-loop level to a basic set of scalar integrals. The application of *TwoCalc* is demonstrated in the following sequence from a *Mathematica* session where the Z boson self-energy amplitude is processed.

(start of *Mathematica* session)

```
> <<TwoCalc.m;
```
(→ loading *TwoCalc* into *Mathematica*)

```
> amp =<<V2V2.amp;
```
(→ the amplitude obtained with *FeynArts* is loaded)

```
> SetOptions[ TwoLoop,
              CollectFunction -> 0];
```
(→ options are set. `CollectFunction` allows to choose between DREG and DRED)

```
> res = TwoLoopSum[amp,
        SelfEnergyPart -> 1];
```
(→ the amplitude is reduced to scalar integrals)

```
> res = Collect[res,
        {A0[_], T[__]}, Simplify];
```
(→ the result is simplified (optional))

```
> res >> V2V2.res;
```
(→ the result for the Z boson self-energy amplitude in terms of scalar integrals is saved in `V2V2.res`)
(end of *Mathematica* session)

The analytically obtained result now consists of scalar integrals at the one- and at the two-loop level. In the case presented here, since the external momentum is set to zero, these are the functions A_0 and T_{134} at the one- and two-loop level, respectively, see Refs. (8, 9).

The pure two-loop diagrams have to be supplemented with the corresponding one-loop diagrams with counter term insertion. In order to obtain the $O(G_F^2 m_t^4)$ corrections an expansion of the amplitudes up to $O(m_t^4/M_W^4)$ had to be performed. All remaining $M_W = M_Z c_W$ have been set to zero. Furthermore we had to apply the MSSM sum rules for Higgs boson masses, especially implying for the case $M_Z, M_W \to 0$ that the lightest MSSM Higgs boson has the mass $m_h = 0$ at tree level. After adding up all contribution, the expansion of the result in terms of $\delta = (4-D)/2$ leads to a finite result in the limit $\delta \to 0$. All relevant details can be found in Ref. (4).

RESULTS FOR $\Delta\rho$ AND M_W

After extracting the prefactor of m_t^4/M_W^4 and setting M_W to zero, besides $s_\beta = \tan\beta/\sqrt{1+\tan^2\beta}$ only two mass scales remain: the top quark mass, m_t, and the mass of the $\mathcal{CP}$-odd Higgs boson, M_A. The result for $\Delta\rho$ can be conveniently expressed in terms of $a \equiv m_t^2/M_A^2$:

$$
\begin{aligned}
\Delta\rho_1^{\mathrm{SUSY}} = 3\,\frac{G_F^2}{128\pi^4}\, m_t^4\, \frac{1-s_\beta^2}{s_\beta^2 a^2} \times \\
\Big\{ \mathrm{Li}_2\left(\left(1-\sqrt{1-4a}\right)/2\right) \frac{8}{\sqrt{1-4a}}\Lambda \\
-2\,\mathrm{Li}_2\left(1-\frac{1}{a}\right)\left[5-14a+6a^2\right] \\
+\log^2(a)\left[1+\frac{2}{\sqrt{1-4a}}\Lambda\right] - \log(a)\Big[2-20a\Big] \\
-\log^2\left(\frac{1-\sqrt{1-4a}}{2}\right)\frac{4}{\sqrt{1-4a}}\Lambda \\
+\log\left(\frac{1-\sqrt{1-4a}}{1+\sqrt{1-4a}}\right)\sqrt{1-4a}(1-2a)
\end{aligned}
$$

$$
\begin{aligned}
&-\log\left(|1/a-1|\right)(a-1)^2 \\
&+\pi^2\left[\frac{2\sqrt{1-4a}}{-3+12a}\Lambda+\frac{1}{3}-2a^2\frac{s_\beta^2}{1-s_\beta^2}\right] \\
&\left.-17a+19\frac{a^2}{1-s_\beta^2}\right\} \qquad (3)
\end{aligned}
$$

with $\Lambda = 3-13a+11a^2$. As a consistency check, in the limit of $M_A \to \infty$, $a \to 0$, we obtain

$$\Delta\rho_1^{\rm SUSY} = 3\,\frac{G_F^2}{128\,\pi^4}\,m_t^4\left[19-2\pi^2\right]. \qquad (4)$$

This is the SM result with $M_H \to 0$. It shows that the MSSM decouples to the SM limit (also at the two-loop level) when the new scale M_A is made large.

With the help of eq. (2) the shift in the W boson mass can be evaluated. The pure SUSY contribution turns out to be small, see Ref. (4). More important, however, is the effective shift from the SM result, which has been used also for the evaluation of the $O(G_F^2 m_t^4)$ corrections within the MSSM (where the SM Higgs boson mass has been set to m_h, the mass of the lightest MSSM Higgs boson), and the new MSSM result, eq. (3). In Fig. 2 this effective change in M_W is shown as a function of m_h.

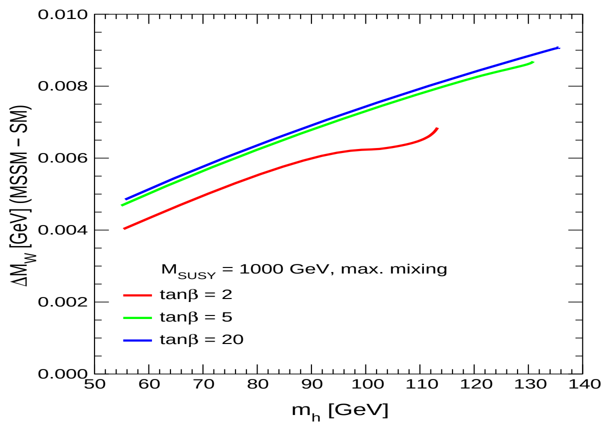

FIGURE 2. The effective change in M_W is shown as a function of m_h for $\tan\beta = 2, 5, 20$.

For the evaluation of m_h in the MSSM we made use of the currently most accurate calculation based on a Feynman-diagrammatic calculation, see Ref. (10), where the result is implemented into the Fortran code *FeynHiggs*, see Ref. (11). The common SUSY mass scale has been set to $M_{SUSY} = 1000$ GeV, the off-diagonal entry in the scalar top mass matrix has been set to $X_t = 2M_{SUSY}$ ("maximal mixing"). The Higgs mixing parameter has been fixed to $\mu = 200$ GeV. For $m_h \gtrsim 90$ GeV, the current LEP2 limit as given in Ref. (12), this results in an effective change in M_W of 5 – 9 GeV.

CONCLUSIONS

We presented the calculation of the SUSY contributions of $O(G_F^2 m_t^4)$ to the ρ-parameter. In order to obtain these two-loop corrections, the computer algebra programs *FeynArts* and *TwoCalc* have been used. Examples of the handling of these programs were given. A compact analytical result in terms of m_t^2/M_A^2 has been derived. The effective change from the previously used SM result to the new MSSM result is an upward shift in M_W of about 5 – 9 GeV.

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