HELAC-PHEGAS: automatic computation of helicity amplitudes and cross sections

Aggeliki Kanaki and Costas G. Papadopoulos

Institute of Nuclear Physics, NCSR Demokritos, 15310 Athens, Greece

Abstract. HELAC-PHEGAS is a `FORTRAN` based package that is able to compute automatically and efficiently tree-level helicity amplitudes and cross sections for arbitrary scattering processes within the standard electroweak theory and QCD. The algorithm for the amplitude computation, HELAC, exploits the virtues of the Dyson-Schwinger equations. The phase-space generation algorithm, PHEGAS, constructs all possible kinematical mappings dictated by the amplitude under consideration. Combined with multichannel self-optimized Monte Carlo integration it results in efficient cross section evaluation.

INTRODUCTION

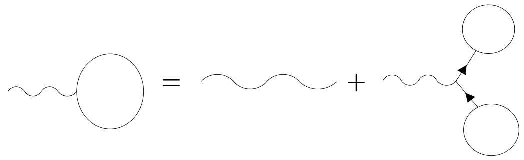
The need for efficient algorithms to calculate helicity amplitudes and cross sections for any process, in an automatic way, has been well recognized long time ago. Up to now, algorithms that efficiently combine helicity amplitude computation and phase-space integration have been proven successful for specific processes, like for instance four-fermion (1) production in e^+e^- collisions. On the other hand general-purpose computational packages like `CompHEP` (2) and `GRACE` (3) do not provide automatic efficient phase-space integration algorithms. Moreover the vast majority of the automatized helicity amplitude computationial algorithms, like for instance `Mad-Graph` (4), have been based on the Feynman graph representation of the amplitude which severely restricts their ability to deal with multiparticle scattering processes.

In this article we report on some developments that have lead to the construction of two programs, HELAC (5) and PHEGAS (6), that allow for an efficient and automatic evaluation of cross sections for arbitrary scattering processes.

HELAC

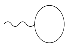
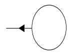
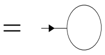
The traditional representation of the scattering amplitude in terms of Feynman graphs results to a computational cost that grows like the number of those graphs, therefore as $n!$ where n is the number of particles involved in the scattering process.

An alternative (7, 8) to the Feynman graph representation is provided by the Dyson-Schwinger approach. Dyson-Schwinger equations express recursively the n-point Green's functions in terms of the $1-, 2-, \ldots, (n-1)$-point functions. For instance in QED these equations can be written as follows:



$$b^\mu(P) = \sum_{i=1}^{n} \delta_{P=p_i} b^\mu(p_i) + \sum_{P=P_1+P_2} (ig)\Pi^\mu_\nu\, \bar\psi(P_2)\gamma^\nu\psi(P_1)\varepsilon(P_1,P_2)$$

where

$b_\mu(P)$ =  $\psi(P)$ =  $\bar\psi(P)$ = 

describes a generic n-point Green's function with respectively one outgoing photon, fermion or antifermion leg carrying momentum P. $\Pi_{\mu\nu}$ stands for the boson propagator and ε takes into account the sign due to fermion antisymmetrization.

In order to actually solve these recursive equations it is convenient to use a binary representation of the momenta involved (8). For a process involving n external particles, all momenta appearing in the computation, P^μ,

$$P^\mu = \sum_{i\in I} p_i^\mu$$

where $I \subset \{1,\ldots,n\}$, can be assigned a binary vector $\vec{m} = (m_1,\ldots,m_n)$, where its components take the values 0 or 1, in such a way that

$$P^\mu = \sum_{i=1}^{n} m_i\, p_i^\mu\ .$$

Moreover this binary vector can be uniquely represented by the integer

$$m = \sum_{i=1}^{n} 2^{i-1} m_i$$

and therefore all sub-amplitudes can be labeled accordingly,i.e.

$$b_\mu(P) \to b_\mu(m)\ ,\ 1 \le m \le 2^{n-1}.$$

A very convenient ordering of integers in binary representation relies on the notion of level l, defined simply as

$$l = \sum_{i=1}^{n} m_i\ .$$

As it is easily seen all external momenta are of level 1, whereas the total amplitude corresponds to the unique level n integer 2^{n-1}. This ordering dictates the natural path of the computation; starting with level-1 sub-amplitudes, we compute the level-2 ones using the Dyson-Schwinger equations and so on up to the level n one which is the full amplitude. For the spinor wave functions as well as for the Dirac matrices, we have chosen the 4-dimensional chiral representation which results to particurarly simple expressions. All electroweak vertices in both the Feynman and the Unitary gauge have been included.

The computational cost of HELAC grows like $\sim 3^n$, which essentially counts the steps used to solve the recursive equations. Obviously for large n there is a tremendous saving of computational time, compared to the $n!$ growth of the Feynman graph approach.

For QCD amplitudes colour representation and summation plays an important role. Let $1 \ldots n$ denote the colour labels of quarks and $\sigma_i(1) \ldots \sigma_i(n)$ denote the colour labels of antiquarks, with $\sigma(i), i = 1 \ldots n!$ being a permutation of $\{1 \ldots n\}$. The colour factor is given obviously by

$$C_i = \delta_{1\sigma_i(1)} \delta_{2\sigma_i(2)} \cdots \delta_{n\sigma_i(n)}$$

Moreover the colour matrix, defined as

$$\mathcal{M}_{ij} = \sum_{\text{colours}} C_i C_j^\dagger$$

with the summation running over all colours, $1 \ldots N_c$, has a very simple representation

$$\mathcal{M}_{ij} = N_c^{m(\sigma_i,\sigma_j)}$$

where $m(\sigma_i,\sigma_j) - 1$ counts how many elements of the permutations σ_i and σ_j are common. In order to extend this colour representation to QCD amplitudes we have to just consider gluons as being quark-antiquark pairs and assign to them two colour labels (i,σ_i). The colour factor and the colour matrix still has exactly the same form. The only thing one has to consider is to rewrite the known Feynman rules of QCD in a slight different way. It is worthwhile to note that exact colour summation is efficient as far as the number of equivalent gluons is smaller than $O(5-6)$. For multicolour processes other approaches have to be considered (9, 10).

The programme is also incorporating the possibility to use an extended precision by exploiting the virtues of `FORTRAN90`. The user can easily switch to a quadruple precision or to an even higher, user-defined precision by using the multi-precision library (11) included in HELAC. In this way, a straightforward computation of cross sections for processes like $e^-e^+ \to e^-e^+e^-e^+$ without any cut is reliably performed (12).

PHEGAS

The study of multi-particle processes, like for instance four-fermion production in e^+e^-, requires efficient phase-space Monte Carlo generators. The reason is that the squared amplitude, being a complicated function of the kinematical variables, exhibits strong variations in specific regions and/or directions of the phase space, lowering in a substantial way the speed and the efficiency of the Monte Carlo integration. A well known way out of this problem relies on algorithms characterized by two main ingredients:

1. The construction of appropriate mappings of the phase space parametrization in such a way that the main variation of the integrand can be described by a set of almost uncorrelated variables, and

2. A self-adaptation procedure that reshapes the generated phase-space density in order to be as much as possible close to the integrand.

In order to construct appropriate mappings we note that the integrand, i.e. the squared amplitude, has a well-defined representation in terms of Feynman diagrams. It is therefore natural to associate to each Feynman diagram a phase-space mapping that parametrizes the leading variation coming from it. To be more specific the contribution of tree-level Feynman diagrams to the full amplitude can be factorized in terms of propagators, vertex factors and external wave functions. In general, the main source of variation comes from the propagator factors and therefore our aim is to construct a mapping that expresses the phase-space density in terms of the kinematical invariants that appear in these propagator factors. Since in principle we need as many mappings as Feynman diagrams for the process under consideration, we have to appropriately combine them in order to produce the global phase-

Table 1. Results for several processes using HELAC-PHEGAS. In the second column the number of Feynman graphs and in parenthesis the number of steps required to solve the recursiveDyson-Schwinger equations are given.

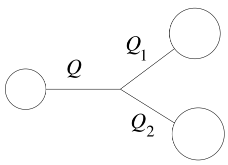
Final states	Number of FG(DS)	$\sqrt{s}$ (GeV)	Cross section (fb)
$e^- e^+ \to u\, \bar{d}\, s\, \bar{c}\, \gamma$	90(74)	200	199.75 (16)
$e^- e^+ \to e^-\, \bar{\nu}_e\, \mu^+\, \nu_\mu\, \gamma$	108(100)	200	29.309 (25)
$e^- e^+ \to \mu^-\, \bar{\nu}_\mu\, u\, \bar{d}\, \gamma\gamma$	587(210)	500	1.730 (58)
$e^- e^+ \to \mu^-\, \bar{\nu}_\mu\, u\, \bar{d}\, c\, \bar{c}$	209(102)	500	0.1783 (20)
$e^- e^+ \to \mu^-\, \bar{\nu}_\mu\, u\, \bar{d}\, c\, \bar{c}\, \gamma$	2142(339)	500	0.02451 (65)
$g\, g \to b\, \bar{b}\, b\, \bar{b}\, W^-\, W^+$	960(380)	500	4.716(24)

space density. A simple and well studied solution to this problem was suggested some time ago in reference (13). It should be mentioned however that other self-adapting approaches can be used as well (14). It is important to note that although by using Feynman graphs to construct phase-space mappings we face the original $n!$ computational cost growth problem, the self-optimization cures this to a certain extent by selecting only the few mappings that dominate the phase-space density. For alternative approaches we refer to (15).

In order to describe the construction of the phase-space mappings, let us consider a typical process in which two incoming particles produce n outgoing ones. The phase space, $d\Phi_n(P = q_1 + q_2; p_1 \ldots, p_n)$, can be decomposed as follows

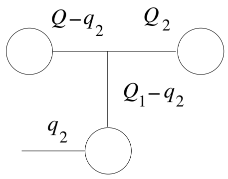
$$d\Phi_n = \left(\prod_{i=1}^{m} \frac{dQ_i^2}{2\pi}\right) d\Phi_m(P; Q_1, \ldots, Q_m)$$
$$d\Phi_{n_1}(Q_1; r_1, r_2, \ldots, r_{n_1}) \ldots d\Phi_{n_m}(Q_m; s_1, s_2, \ldots, s_{n_m})$$

where the subsets $\{r_1, r_2, \ldots, r_{n_1}\}$ up to $\{s_1, s_2, \ldots, s_{n_m}\}$ represent an arbitrary partition of $\{p_1, p_2, \ldots, p_n\}$. The above equation can be generalized recursively resulting in an arbitrary decomposition of $d\Phi_n$. Feynman graphs can be seen as a realization of such a decomposition, this latter being identified with a sequence of vertices of the graph. There are two possible cases for $2 \to n$ scattering. First, all outgoing momenta involved in the vertex are time-like,



$$\begin{aligned} d\Phi_n &= \ldots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} d\Phi_2(Q \to Q_1, Q_2) \ldots \\ &= \ldots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} d\cos\theta\, d\phi\, \frac{\lambda^{1/2}(Q^2, Q_1^2, Q_2^2)}{32\pi^2\, Q^2} \ldots \end{aligned}$$

with $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, and second when one of them is space-like,



$$\begin{aligned} d\Phi_n &= \ldots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} d\Phi_2(Q \to Q_1, Q_2) \ldots \\ &= \ldots \frac{dQ_1^2}{2\pi} \frac{dQ_2^2}{2\pi} dt\, d\phi\, \frac{1}{32\pi^2\, Q\, |\vec{q}_2|} \ldots \end{aligned}$$

with

$$\begin{aligned} t &= (Q_1 - q_2)^2 \\ &= m_2^2 + Q_1^2 - \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2) + \frac{\lambda^{1/2}}{Q} |\vec{q}_2| \cos\theta \end{aligned}$$

and $(E_2, \vec{q}_2)$ being the incoming momentum q_2 in the rest frame of Q. The appropriate sequence of vertices, $\{V_1, V_2, \ldots, V_k\}$ can be chosen in such a way that a recursive construction of the phase space is realized. For instance V_1 should contain at least one incoming particle whose momentum is known. The rest of the sequence is chosen recursively: vertex V_j is characterized by an incoming momentum Q which has already been generated in one of the $\{V_1, \ldots, V_{j-1}\}$.

Following the above described algorithm we end up with an expression for the phase-space density,

$$d\Phi_n \to \prod ds_i \mathcal{P}_i(s_i) \prod dt_j \mathcal{P}_j(t_j) \prod d\phi_k \prod d\cos\theta_l$$

where s_i and t_j refer to the kinematical invariants entering the propagator factors of the graph and ϕ_k and $\cos\theta_l$ represent center-of-mass angles needed to complete the phase space parametrization. It is now straightforward to generate s_i and t_j with probability densities $\mathcal{P}_i(s_i)$ and $\mathcal{P}_j(t_j)$ that are automatically chosen accordingly to the nature of the propagating particle.

Results, demonstrating the ability of HELAC-PHEGAS to deal with multiparticle processes, are presented in table 1 (6, 16).

SUMMARY AND OUTLOOK

HELAC-PHEGAS offers a framework for high-energy phenomenology. It provides all necessary and sufficient tools for efficient, reliable and automatic computation of helicity amplitudes and cross sections. The Standard Model, including QCD, has been fully incorporated. Higher-order corrections are in principle tractable within the framework of Dyson-Schwinger equations and work is in progress in order to include electroweak corrections as described in reference (17). New physics interactions and models, icluding the Minimal Supersymmetric Standard Model and the trilinear gauge couplings will be considered in the near future.

Acknowledgments
C.G.P. would like to acknowledge Fermilab and Argonne National Laboratory for their kind hospitality.

REFERENCES

1. D. Bardin *et al.*, "Event generators for W W physics," hep-ph/9709270 and references therein.
2. E. E. Boos, M. N. Dubinin, V. A. Ilin, A. E. Pukhov and V. I. Savrin, "CompHEP: Specialized package for automatic calculations of elementary particle decays and collisions," hep-ph/9503280.
   V. A. Ilin, D. N. Kovalenko and A. E. Pukhov, Int. J. Mod. Phys. **C7** (1996) 761 [hep-ph/9612479].
3. T. Ishikawa, T. Kaneko, K. Kato, S. Kawabata, Y. Shimizu and H. Tanaka [MINAMI-TATEYA group Collaboration], KEK-92-19.
   F. Yuasa *et al.*, "Automatic computation of cross sections in HEP: Status of GRACE system," hep-ph/0007053.
4. T. Stelzer and W. F. Long, Comput. Phys. Commun. **81** (1994) 357 [hep-ph/9401258].
5. A. Kanaki and C. G. Papadopoulos, "HELAC: A package to compute electroweak helicity amplitudes," hep-ph/0002082.
6. C. G. Papadopoulos, "PHEGAS: A phase space generator for automatic cross-section computation," hep-ph/0007335.
7. F. A. Berends and W. T. Giele, Nucl. Phys. **B306** (1988) 759.
8. F. Caravaglios and M. Moretti, Phys. Lett. **B358** (1995) 332 [hep-ph/9507237].
9. P. Draggiotis, R. H. Kleiss and C. G. Papadopoulos, Phys. Lett. **B439** (1998) 157 [hep-ph/9807207].
10. F. Caravaglios, M. L. Mangano, M. Moretti and R. Pittau, Nucl. Phys. **B539** (1999) 215 [hep-ph/9807570].
11. David M. Smith, Transactions on Mathematical Software **17** (1991) 273- 283. http://www.lmu.edu/acad/personal/faculty/dmsmith2/FMLIB.html
12. F. A. Berends, C. G. Papadopoulos and R. Pittau, "NEXTCALIBUR: A four-fermion generator for electron positron collisions," hep-ph/0011031; "Four-fermion production in electron positron collisions with NEXTCALIBUR," hep-ph/0002249.
13. R. Kleiss and R. Pittau, Comput. Phys. Commun. **83** (1994) 141 [hep-ph/9405257].
14. T. Ohl, Comput. Phys. Commun. **120** (1999) 13 [hep-ph/9806432].
15. P. D. Draggiotis, A. van Hameren and R. Kleiss, Phys. Lett. **B483** (2000) 124 [hep-ph/0004047].
16. M. W. Grunewald *et al.*, "Four fermion production in electron positron collisions," hep-ph/0005309.
17. W. Beenakker, F. A. Berends and A. P. Chapovsky, Nucl. Phys. **B573** (2000) 503 [hep-ph/9909472].