Analytical calculation of heavy baryon correlators in NLO of perturbative QCD

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Abstract. We present analytical results for the correlator of baryonic currents at the three-loop level with one finite mass quark. We obtain the massless and the HQET limits as particular cases from the general formula. Calculations have been performed with an extensive use of the symbolic manipulation programs MATHEMATICA and REDUCE.

Baryons form a rich family of particles which has been experimentally studied with high accuracy [1]. A theoretical analysis of these experimental data gives a lot of information about the structure of QCD and the numerical values of its parameters. The hypothetical limit $N_c \to \infty$ for the number N_c of colours which is a very powerful tool for investigating the general properties of gauge interactions was especially successful for baryons [2]. The spectrum of baryons is contained in the correlator of two baryonic currents and the spectral density associated with it. To leading order the correlator is given by a product of N_c fermionic propagators. The diagrams of this topology have recently been studied in detail [3, 4, 5, 6, 7, 8]. They are rather frequently used in phenomenological applications [9]. With the advent of new accelerators and detectors many properties of baryons containing a heavy quark have been experimentally measured in recent years [1]. However, theoretical calculations beyond the leading order have not been done for many interesting cases. In this note we fill up this gap.

We report on the results of calculating the α_s corrections to the correlator of two baryonic currents with one finite mass quark and two massless quarks. We give analytical results and discuss the magnitude of the α_s corrections. The massless and HQET limits are obtained as special cases. Note that the massless case has been known since long ago [10]. The mesonic analogue of our baryonic calculation was completed some time ago [11] and has subsequently provided a rich source of inspiration for many applications in meson physics.

A generic baryonic current has the form

$$j = \varepsilon^{abc}(u_a^T C d_b)\Gamma\Psi_c \tag{1}$$

where Γ is the Dirac matrix, in the following $\Gamma = 1$.

The correlator of two baryonic currents is expanded as

$$i\int \langle T\, j(x)\,\bar{j}(0)\rangle e^{iqx}dx = \gamma_\nu q^\nu \Pi_q(q^2) + m\Pi_m(q^2). \tag{2}$$

Here we show results for the function $\Pi_q(q^2)$ and compare it with $\Pi_m(q^2)$ [12]. The dispersion relation reads

$$\Pi_\#(q^2) = \frac{1}{128\pi^4}\int_{m^2}^{\infty}\frac{\rho^\#(s)ds}{s-q^2} \tag{3}$$

where $\rho^\#(s) = \rho^{q,m}(s)$ are the spectral densities. The spectral density is the real object of interest for phenomenological applications,

$$\rho^\#(s) = s^2\left\{\rho_0^\#\left(1+\frac{\alpha_s}{\pi}\ln\left(\frac{\mu^2}{m^2}\right)\right)+\frac{\alpha_s}{\pi}\rho_1^\#\right\}. \tag{4}$$

Here μ is the renormalization scale parameter, m is a pole mass of the heavy quark (see e.g. Ref. [13]) and $\alpha_s = \alpha_s(\mu)$. The leading order two-loop contribution is shown in Fig. 1(a). This topology coincides with water melon diagrams for which a general method of calculation (with arbitrary masses) has recently been developed [5, 6, 7]. The leading order results read

$$\begin{aligned}\rho_0^q &= \tfrac{1}{4} - 2z + 2z^3 - \tfrac{1}{4}z^4 - 3z^2\ln z, &(5)\\ \rho_0^m &= 1 + 9z - 9z^2 - z^3 + 6z(1+z)\ln z &(6)\end{aligned}$$

with $z = m^2/s$. The next-to-leading order contribution is given by three-loop diagrams with one external momentum. For an arbitrary mass arrangement such diagrams have not yet been calculated analytically. However, if we take the case of one massive line, the result within $\overline{\rm MS}$-scheme can be obtained analytically and reads

$$\begin{aligned}\rho_1^q = {} & \tfrac{71}{48} - \tfrac{565}{36}z - \tfrac{7}{8}z^2 + \tfrac{625}{36}z^3 - \tfrac{109}{48}z^4\\ & -\left(\tfrac{49}{36} - \tfrac{116}{9}z + \tfrac{116}{9}z^3 - \tfrac{49}{36}z^4\right)\ln(1-z)\\ & +\left(\tfrac{1}{4} - \tfrac{17}{3}z - 11z^2 + \tfrac{113}{9}z^3 - \tfrac{49}{36}z^4\right)\ln z\\ & +\left(\tfrac{1}{3} - \tfrac{8}{3}z + \tfrac{8}{3}z^3 - \tfrac{1}{3}z^4\right)\ln(1-z)\ln z\\ & -2z^2\left(9 + \tfrac{4}{3}z - \tfrac{1}{6}z^2\right)\left(\tfrac{1}{2}\ln^2 z - \zeta(2)\right)\\ & +\left(\tfrac{2}{3} - \tfrac{16}{3}z - 18z^2 + \tfrac{8}{3}z^3 - \tfrac{1}{3}z^4\right){\rm Li}_2(z)\\ & -12z^2\left({\rm Li}_3(z) - \zeta(3) - \tfrac{1}{3}{\rm Li}_2(z)\ln(z)\right) \qquad (7)\end{aligned}$$

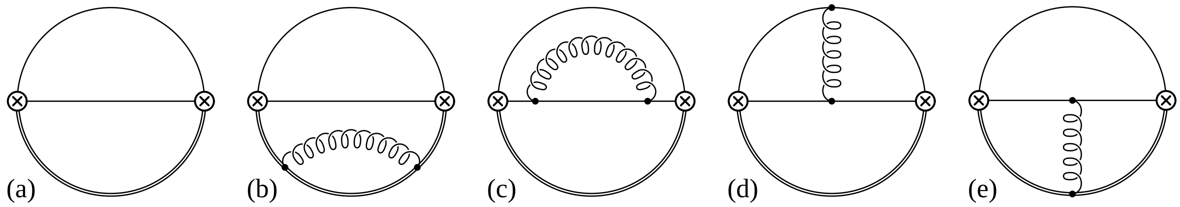

FIGURE 1. The calculated (a) two-loop and (b–e) three-loop topologies

The contributing three-loop diagrams are shown in Figs. 1(b) to (e). They have been evaluated using advanced algebraic methods for multi-loop calculations along the lines decribed in Refs. [6, 11]. This result should be compared to

$$\begin{aligned}\rho_1^m \;=\; & 9+\tfrac{665}{9}z-\tfrac{665}{9}z^2-9z^3 \\ & -\left(\tfrac{58}{9}+42z-42z^2-\tfrac{58}{9}z^3\right)\ln(1-z) \\ & +\left(2+\tfrac{154}{3}z-\tfrac{22}{3}z^2-\tfrac{58}{9}z^3\right)\ln z \\ & +4\left(\tfrac{1}{3}+3z-3z^2-\tfrac{1}{3}z^3\right)\ln(1-z)\ln z \\ & +12z\left(2+3z+\tfrac{1}{9}z^2\right)\left(\tfrac{1}{2}\ln^2 z-\zeta(2)\right) \\ & +4\left(\tfrac{2}{3}+12z+3z^2-\tfrac{1}{3}z^3\right)\mathrm{Li}_2(z) \\ & +24z(1+z)\left(\mathrm{Li}_3(z)-\zeta(3)-\tfrac{1}{3}\mathrm{Li}_2(z)\ln z\right). \quad (8)\end{aligned}$$

Our method of integration is a completely algebraic one and therefore symbolic manipulation programs can be used for performing the long calculations. Two independent calculations of some steps were done using MATHEMATICA and REDUCE, the latter being rather actively used for high energy calculations (see e.g. Ref. [14]).

The results given in Eqs. (7) and (8) represent the full next-to-leading order solution. Since the anomalous dimension of the current in Eq. (1) is known up to two-loop order [15], the results shown in Eqs. (7) and (8) complete the ingredients necessary for an analysis of the correlator in Eq. (2) within operator product expansion at the next-to-leading order level.

Two limiting cases of general interest are the near-threshold and the high energy asymptotics. With our result given in Eq. (7) both limits can be taken explicitly.

In the massless limit $z\to 0$ the corrections read

$$\rho_1^q \;=\; \tfrac{71}{48}+\tfrac{1}{4}\ln z-\tfrac{41}{3}z-6z\ln z+\ldots, \quad (9)$$

$$\rho_1^m \;=\; 9+83z-4\pi^2 z+2\ln z+50z\ln z+\ldots. \quad (10)$$

Therefore we obtain

$$\rho^q(s)=\frac{s^2}{4}\left\{1+\frac{\alpha_s}{\pi}\left(\ln\left(\frac{\mu^2}{s}\right)+\frac{71}{12}\right)\right\}$$
$$-2m^2_{\overline{\rm MS}}(\mu)s\left\{1+\frac{\alpha_s}{\pi}\left(3\ln\left(\frac{\mu^2}{s}\right)+\frac{19}{2}\right)\right\}, \quad (11)$$

$$m\rho^m(s)=m_{\overline{\rm MS}}(\mu)s^2\left\{1+\frac{\alpha_s}{\pi}\left(2\ln\left(\frac{\mu^2}{s}\right)+\frac{31}{3}\right)\right\}. \quad (12)$$

For the momentum part $\rho^q(s)$ we retain the $O(m^2)$ correction. The relation between the pole mass m and the $\overline{\rm MS}$ mass $m_{\overline{\rm MS}}(\mu)$ we have used reads

$$m=m_{\overline{\rm MS}}(\mu)\left\{1+\frac{\alpha_s}{\pi}\left(\ln\left(\frac{\mu^2}{m^2}\right)+\frac{4}{3}\right)\right\}. \quad (13)$$

In the near-threshold limit $E\to 0$ with $s=(m+E)^2$ one explicitly obtains

$$\rho^m_{\rm thr}(m,E) \;=\; \frac{16E^5}{5m}\left\{1+\frac{\alpha_s}{\pi}\ln\left(\frac{\mu^2}{m^2}\right)\right. \quad (14)$$
$$\left.+\frac{\alpha_s}{\pi}\left(\frac{54}{5}+\frac{4\pi^2}{9}+4\ln\left(\frac{m}{2E}\right)\right)\right\}+O\left(\frac{E^6}{m^2}\right).$$

We have shown the coincidence with the result of the explicit HQET calculation,

$$m\rho^m_{\rm thr}(m,E)=C(m/\mu,\alpha_s)^2\rho_{\rm HQET}(E,\mu) \quad (15)$$

where $\rho_{\rm HQET}(E,\mu)$ and $C(m/\mu,\alpha_s)$ with

$$\rho_{\rm HQET}(E,\mu) \;=\; \frac{16E^5}{5}\left\{1+\frac{\alpha_s}{\pi}\left(\frac{182}{15}+\frac{4\pi^2}{9}+4\ln\frac{\mu}{2E}\right)\right\}$$
$$C(m/\mu,\alpha_s) \;=\; 1+\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln\left(\frac{m^2}{\mu^2}\right)-\frac{2}{3}\right) \quad (16)$$

are taken from Refs. [16, 17], respectively. Note that the higher order corrections in E/m to Eq. (14) can easily be obtained from the explicit result given in Eq. (7).

Of interest is whether the two limiting expressions (the massless limit expression as given in Eq. (11) and the HQET limit expression in Eqs. (14) and (15)) can be used to characterise the full function for all energies.

For this discussion we compare components of the baryonic spectral function in leading and next-to-leading order. In Fig. 2 and 3 we show the ratio $\rho^\#_1(s)/\rho^\#_0(s)$ for $\# = m$ and $\# = q$, respectively. In the following we shall always use the specific renormalization scale value $\mu=m$ if it is not written explicitly. One can see that a simple interpolation between the two limits can give a rather good approximation for the next-to-leading order correction in the complete region of s. We therefore conclude that in going even one order higher it is very likely

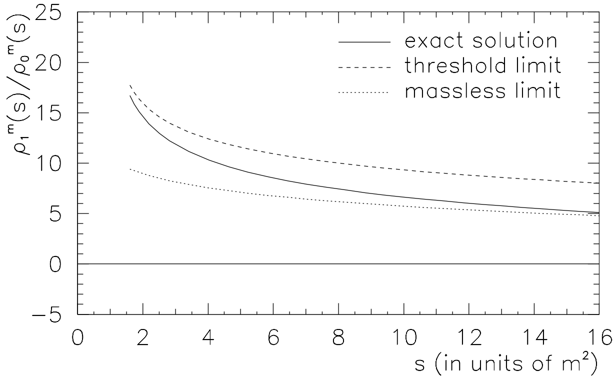

FIGURE 2. The ratio ρ_1^m/ρ_0^m in dependence of s

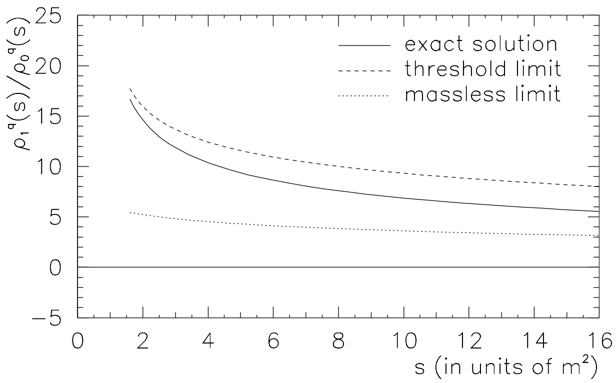

FIGURE 3. The ratio ρ_1^q/ρ_0^q in dependence of s

that the full four-loop spectral density can be well approximated by the corresponding massless four-loop result which can be calculated using existing computational algorithms [18, 19].

ACKNOWLEDGEMENTS

We want to thank Richard Kreckel for presenting our poster at the conference. The present work is supported in part by the Volkswagen Foundation under contract No. I/73611 and by the Russian Fund for Basic Research under contract 99-01-00091. A.A. Pivovarov is an Alexander von Humboldt fellow. S. Groote gratefully acknowledges a grant given by the DFG, FRG.

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