Summaries of Recent Computer-assisted Feynman Diagram Calculations

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Summarizing results submitted by

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Abstract. Recent results from several researchers, in the area of automated high-order computation of quantities in QCD, the Standard Model, and other models are summarized.

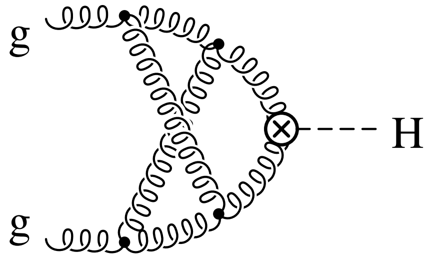
The AIHENP Workshop series has traditionally included cutting edge work on automated computation of Feynman diagrams. The conveners of the Symbolic Problem Solving topic in this ACAT conference felt it would be useful to solicit presentations of brief summaries of the interesting recent calculations. Since this conference was the first in the series to be held in the Western Hemisphere, it was decided that the summaries would be solicited both from attendees and from researchers who could not attend the conference. This would represent a sampling of many of the key calculations being performed. The results were presented at the Poster session; contributions from ten researchers were displayed and posted on the web.

Although the poster presentation (which can be viewed at *conferences.fnal.gov/acat2000/* placed equal emphasis on results presented at the conference and other contributions, here we primarily discuss the latter, which do not appear in full form in these proceedings.

This brief paper can't do full justice to each contibution; interested readers can find details of the work not presented at this conference in references (1), (2), (3), (4), (5), (6), (7).

Standard Model Higgs Prodution

Robert Harlander has results(1) for $gg \longrightarrow H$ to two loops (NNLO) in the heavy top limit. This will be the dominant production mechanism for the Higgs at the LHC, so it is important to improve on the theoretical accuracy. At next-to-leading-order, the theoretical uncertainty



Source: R. Harlander(1)

FIGURE 1. Sample two-loop diagram contributing to $gg \longrightarrow H$ at NNLO.

is a factor of 1.5 to 2. The NNLO contributions include diagrams like that shown in figure 1.

When $M_H < M_t$ an expansion in M_H^2/M_t^2 yields an excellent approximation at NLO) (and presumedly at NNLO). The leading term in this expansion may be obtained by using effective Lagrangian for the Higgs-gluon interaction. The coefficient of $H(G_{\mu\nu})^2$ for the effective vertex (marked $\otimes$ in the figure) was previously computed to the needed order in α_S(9).

The NNLO corrections sum several contributions, for example, one-loop amplitudes involving radiation of a single quark or gluon. Some (but not all) of these have been determined. The Harlander calculation computes contribution of the gauge invariant set of corrections (of order α_S^4) involving two loops and no extraneous radiation.

The diagrams which are planar had been reduced, using an integration-by-parts algorithm, to convolutions of one-loop integrals(11). Non-planar diagrams such as the one shown are, as usual, less straightforward. The technique used was one developed by Baikov and Smirnov(10): The recurrence relations for these 2-loop integrals with 3 external legs are related to those for 3-loop integrals with two external legs. Thus such diagrams as Fig. 1 are mapped onto massless three-loop two-point functions.

These calculations were done, using a modification of the program MINCER(13) which is written in the symbolic manipulation system FORM(16). Programmed in this manner, the computer calculation was not very extensive: It completed in a few minutes on a fast processor.

The primary result(1) is a second order correction to the virtual cross section for $gg \longrightarrow H$. As an estimate on the magnitude of the corrections, the ratio of time-like to space-like form factor is considered. For 5 light (on the scale of M_H) quarks, the NLO correction to this was 52.8%. The newly computed NNLO correction is found to be 17.2% (and in the same direction). Thus the correction is large, but there is good convergency.

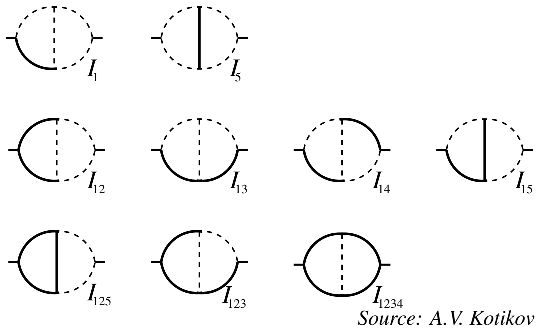
Advanced mathematical techniques and high-order diagrams

A.V. Kotikov(2) uses a Differential Equations Method to do two-loop self-energy diagrams with one non-zero internal mass or external momentum.Combinations of DEM and programs by O.L. Veretin and M. Kalmykov have been used to evaluate that full set of two-loop, two-point onshell master diagrams, as well as three-point two-loop integrals with one and two-mass thresholds in a small-moment expansion.

The Differential Equations Method makes use of the integration-by-parts method(15): when applied to an internal n-point subgraph of a Feynman diagram, IBP generates new diagrams which can be represented as derivatives with respect to masses (or external momenta) of the initial diagram. Thus a differential equation for the initial diagram can be found; this equation has inhomogeneous terms containing diagrams with more trivial topological structure and/or fewer loops or legs. Complicated diagrams may be evaluated by recursively applying this procedure to reduce to known results for simpler diagrams. Some results can be found in reference (2).

In the summary posted at the conference results for two-loop self-energy diagrams in figure 2 were presented. For example,

$$q^2 \cdot I_{125} = -2\log^2 y \log(1-y) - 6\zeta_3 + 6\mathrm{Li}_3(y) - 6\log y \mathrm{Li}_2(y)$$



Source: A.V. Kotikov

FIGURE 2. Two-loop self-energy diagrams. Solid lines denote propogators with mass m; dashed lines denote massless propagators.

where

$$y \equiv \frac{1-\sqrt{q^2/(q^2-4m^2)}}{1+\sqrt{q^2/(q^2-4m^2)}}$$

A.L. Kataev, G. Parente and A.V. Sidorov sent results(3) of using the method of Jacobi polynomials to to a next-to-next-to-leading order analysis of Fermilab data for the xF_3 structure function of νN deep-inelastic scattering. Using analytical expressions(14) for the theoretical behaviour of QCD (including $\alpha_S(M_Z)$ and higher-twist terms), these researchers along with A.V.Kotikov(4) use a FORTRAN program realizing the Jacobi Polynomial method, to fit the data. This results in values of $\Lambda_{\overline{MS}}$ normalized to the x-behavior of the nonperturbative contribution, modeled as $h(x)/Q^2$.

They observe that using the Jacobi polynomial method it is possible to reconstruct the structure function to rather high precision, using only ten of its Mellin moments. An interesting physics point is that there is interplay between the effects of the NNLO perturbative QCD corrections and $1/Q^2$-contributions, which result in effective "shadowing" of the power-supressed terns by the perturbative NNLO effects. If the $1/Q^2$-contributions are fixed through a special model, the NNLO value of $\alpha_S(M_Z)$ is $0.118 \pm 0.002(\text{stat}) \pm 0.005(\text{syst}) \pm 0.003(\text{theory})$.

S. Eidelman, F. Jegerlehner, A.L. Kataev, and O.V. Veretin(5) contributed results of three-loop massive corrections to the Adler D-function of the e^+e^- annihilation process. These were calculated in the Euclidean region, using a Padé resummation method(12). Massive diagrams with one external momentum were considered, with the Padé resummations realized in a custom FORTRAN program. These were run in several minutes on an Alpha workstation.

At high energies, the perturbative QCD prediction starts to agree with the experimentally motivated behaviour of the Adler D-function only after inclusion of the mass dependence of this 3-loop order α_S^2 term. Thus, these results allow extraction of hadronic shifts to the fine structure constant, from experimental data on the cross section for annihilation of e^+e^- into hadrons, including data obtained at the low-enery e^+e^- in Novosibursk.

D.J. Broadhurst, A.L. Kataev and C.J. Maxwell(6) report on large N_f expansion of scalar correlators and estimates of higher-order QCD corrections to Higgs $\longrightarrow \bar{b}b$ and strange-quark-mass sum rules. These large N_f terms come from a single chain of quark bubble diagrams, and the two-point correlator of the scalar quark current $\bar{\psi}\psi$ was calculated to 20 loops analytically, and up to 100 loops numerically. Such correlators are related to the decay width of the scalar Higgs into quark-antiquark pairs.

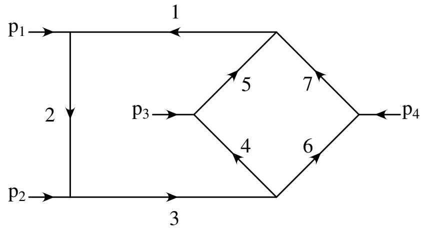
The n-loop diagrams were calculated by inserting $n-2$ quark loops in the pair of two-loop skeletons for the scalar correlator. The method entails recurrence relations for $_3F_2$ hypergeometric series, which were implemented in REDUCE. The analysis demonstrates that one must take a twice-cubtracted dispersion relation to avoid an ambiguity of order Λ^2/Q^2 even in the zero-quark-mass limit. Failure to do so leads to explosion of the perturbation series.

Estimates are obtained for order α_S^4 contributions, paying particular attention to terms that result from analytic continuation, which are resummed to all orders in α_S and to leading order in β_0. It is concluded that the perturbative uncertainties in the extraction of strange quark mass are mild, compared with uncertainties related to poor knowledge of the low-enery hadronic spectral function.

C. Oleari, contributed results(8) obtained in collaboration with C. Anastasiou, E.W.N. Glover and M.E. Tejeda-Yeomans, on two-loop QCD corrections to $q\bar{q} \longrightarrow q'\bar{q}'$. They associate tensor integrals with scalar integrals in higher dimension and with higher powers of propagators, by using the Schwinger-parameter form. Systematic application of the Integration-By-Parts technique, along with recursion relations, is sufficient to reduce these integrals to master intergrals in $D = 4-2\varepsilon$. With four external quark lines, the most challenging two-loop topology is the "crossed box" diagram:

The IBP and recursion techniques for reducing these two-loop, four-leg diagrams to master integrals (and in particular integrals for the massless crossed box topology) are discussed in Oleari's parallel session presentation at this conference(17).

They have used these identities to construct MAPLE, MAXIMA, and FORM programs to rewrite the tensor integrals for massless $2 \longrightarrow 2$ scattering directly in terms of the basis set of master integrals.



Source: C. Oleari(17)

FIGURE 3. The generic two-loop crossed box.

Then amplitudes are computed by generating the one- and two-loop diagrams using QGRAF. After projecting by tree level and summing over colours, spins and Dirac traces in D dimensions, they identify the scalar and tensor integrals present and replace them with combinations of master integrals. These are then expanded in ε. The expansion can be broken into two parts: one proportional to the Born amplitude $\mathcal{A}^4$, and one that depends on kinematic structures that do not occur at the tree level. The detailed results are rather lengthy for presentation in a summary paper such as this, and can be examined in (8).

Calculations in the MSSM Model

S. Heinemeyer presented work(7) done with G. Weiglein, the developer of the MATHEMATICA program *TwoCalc*. They used that program, along with *FeynArts* (developed by T. Hahn) to compute electroweak two-loop corrections in the Minimal Supersymmetric Standard Model.

The physics motivation is that SUSY particles are too heavy to directly observe in today's colliders, so one must search for indirect effects, by looking at precision observables. The electroweak precision data can be compared with theoretical predictions of the Standard Model and MSSM, to see which model fits better and potentially contradict one or the other. But this tests the theory at the quantum level, and is sensitive to loop corrections. Very high accuracy of measurements and theoretical predictions are needed. In particular, two-loop calculations are necessary to achieve this accuracy on the theoretical side.

The ρ-parameter gives the main contribution to correctins to electroweak observables such as M_W and $\sin^2\theta_W^{\text{eff}}$, and the leading two-loop corrections in MSSM to $\Delta\rho$, which are of order $G_F^2 m_t^4$, are comparable to the accuracy obtained in the Standard Model and to prospective experimental uncertainties. The two-loop results for $\Delta\rho_1^{\text{SUSY}}$ are given in (7).

The contribution of these two-loop diagrams to ΔM_W(MSSM–SM) depends on the Standard Model

$\tan\beta$, and on m_h or M_A (which are related in MSSM). Its dependence on m_h is presented in (7) for several values of $\tan\beta$, and the poster summary displays the dependence of ΔM_W on $\tan\beta$, for several values of M_A.

Beyond the large number of diagrams involved in this computation, the probelm of a proliferation of scales in the MSSM further complicates evaluation of the two-loop corrections. The computation made heavy use of MATHEMATICA-based computer algebra programs: *FeynArts* to generate Feynman diagrams and amplitudes, and *TwoCalc* for reduction of tensor integrals to scalar integrals and evaluation of those integrals. The computing time amounted to about a day on a 500 MHz Pentium.

Results Presented in Parallel or Poster Sessions

The Feynman calculation summary poster was intended to be inclusive: neither all extra-conference material, nor all in-conference presentations. The following summaries were provided based on work presented at this conferences; the detailed papers can of course be found in these proceedings:

S. Groote and A.A. Pivovarov submitted results of a calculation of three-loop QCD diagrams for massive baryonic correlators. These are next-to-leading-order calculations for such processes. Such diagrams are ingredients for QCD sum rules which aim to determine basic baryonic quantities like gorund state energy or residues. The massless contributions were comuputed using REDUCE and MATHEMATICA in a few minutes of computing time; the massive contributions were done by hand and took a few days. These results were persneted as a poster session by R. Kreckel, and appear, co-authored by J.G. Körner, in these proceedings(18).

E.E. Boos presented a scheme(19) for finding gauge-invariant subclasses of diagrams for a given process. For example, in Bhabha-scattering, the two s-channel and two t-channel SM diagrams are separately gauge invariant. Aside from the advantage of being able to deal with smaller pieces of a difficult calculation and still produce physically meaningful numbers, this subclassing scheme is important because the precision of computation can be helped by the freedom to use different kinematical variables of integration for different susbsets of diagrams.

F. Yuasa, T. Kaneko and T. Ishikawa presented the Feynman graph selection tool (`grcsel`) in the GRACE system. GRACE(20) is a collection of tools which provides automated generation of Feynman graphs and corresponding helicity amplitudes, phase space integration of the squared amplitudes, and event generation for data analysis. It also contains a facility GRACEFIG for generating figures containing the generated diagrams. The `grcsel` tool(21) can handle tree and 1-loop graphs and supports the Standard and MSSM Models.

State of Computer Techniques

This sample of leading-edge work can provide a perspective on the way advanced computing techniques are being applied to the difficult problem of high-order Feynman graph calculation. Four observations:

- The state of the art has long since passed the point where you could consider doing all these calculations by hand, though there are still some important calculations which have not been done, yet which are not so large as to absolutely require computer assistance.
- There is no clearly established prefered symbolic manipulation system for Feynman integral calculations. Of the ten summaries submitted, five systems (FORM, REDUCE, MATHEMATICA, MACSYMA, MAPLE, and QGRAF) were utilized for calculations, and only one (MATHEMATICA) was used in two cases. Three major programs within systems (*Mincer*, *FeynArts*, *TwoCalc*, three specialized FORTRAN programs, and a major framework GRACE for the non-integration parts of the problem were also used.
- Not everything that could be automated was done via computer. There were a couple of cases where one class of diagrams or one major step was done by hand. This reflects either continuing difficulty in expressing to an automated system the steps done by hand, or a lack of faith that the automated expressions of these steps would be executed correctly. Here, there is room for improvement in the computer tools available.
- Surprisingly, none of the calculations occupied a significant amount of computer time. The running times were generally several minutes and ranged up to one day, and there was no temptation to use any computing platform beyond a simple workstation.

This last observation leads to the conclusion that in principle, the Symbolic Probelm Solving community has the hardware and the software frameworks to attack substantially more complex problems than are currently being pursued. Instead of computing power, one limiting factor (and an area where work being done today will fundamentally advance the field) is continuing development of mathematical techniques and physical insights to do (and to organize) higher-loop calculations with several distinct masses and external momenta. And a related opportunity for improvement is in tools to comfortably program those sophisiticated mathematical techniques in a reliable and readable manner.

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