Mass dependence of the QCD contributions to the Adler function: theory vs experiment

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*) The three-loop massive corrections to the Adler D-function of the e^+e^--annihilation process were calculated in the Euclidean region using the Padé resummation method, developed in the papers by D.J. Broadhurst, J. Fleischer and O.V. Tarasov, *Z. Phys.* **C60** (1993) 287 and J. Fleischer and O.V. Tarasov, *Z. Phys.* **C64** (1994) 413. This method is based on the expansion of Feynman diagrams in powers of m^2/Q^2 and Q^2/m^2, where Q^2 is the Euclidean momentum transfer, with further application of the conformal mapping and Padé resummation of the convergent massive-expanded series.

As an input for our analysis, we took the analytical expressions for the coefficients of the expansion at low and high Q^2 calculated using the FORM analytical system by the members of Karlsruhe group (References: K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, *Nucl. Phys.* **B482** (1996) 213; *Nucl. Phys.* **B505** (1997) 40; K.G. Chetyrkin et al.,*Nucl. Phys.* **B503** (1997) 339).

*) The tree-loop massive diagrams with one external momentum, contributing to the photon vacuum polarization function, were considered.

*) Padé resummation of the mass-expanded 3-loop diagrams was realized in the form of the specially written FORTRAN programs.

The phenomenological part of the Adler D-function was calculated using the special FORTRAN package, written down in the process of the extraction of the hadronic contributions to the shift of the fine structure constant from the available experimental data on the cross section of e^+e^--annihilation into hadrons including those obtained at the low-energy e^+e^--collider in Novosibirsk (see S.I. Eidelman, F. Jegerlehner, *Z. Phys.* **C67** (1995) 585; F. Jegerlehner, *Nucl. Phys.Proc. Suppl.* **B51C** (1996) 131).

*) The calculation of the mass dependent 3-loop corrections was done at the Alpha station, the typical running time is few seconds.

The experimentally motivated behaviour of the e^+e^--annihilation D-function in the Euclidean region was obtained at the Alpha station, with a typical running time of few minutes.

*) Reference: S. Eidelman, F. Jegerlehner, A.L. Kataev and O. Veretin, *Phys. Lett.* **B454** (1999) 369

*) Abstract was not presented at ACAT2000.

*) Motivation: The Adler function is defined as

$$D(-s) = \frac{3\pi}{\alpha} s \frac{d}{ds} \Delta\alpha_{\rm had}(s) = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R^{e^+e^-}(s)}{(s+Q^2)^2} ds$$

where $R^{e^+e^-}$ is proportional to the $e^+e^- \to harons$ total cross-section and $\alpha_{\rm had}(s)$ is the shift of the fine structure constant due to the hadronic contribution. We have a good opportunity to test pertubartive QCD at higher orders using this function, evaluated in two different ways: at the quark level using Feynman diagrams and as the dispersion integral, using experemental data on e^+e^--annihilation into hadrons. In the Euclidean region this function is smooth, since the resonance and threshold effects in $R^{e^+e^-}(s)$, which are typical to the Minkowskian region, are suppressed in the dispersion integral by the weight function $Q^2/(s+Q^2)^2$. Moreover, the application of the renormalization group is transparent here. Indeed, it does not require the additional consideration of the proportional to π^2-effects of analytical continuation of the coefficients of multiloop perturbative series from the Euclidean to the Minkowskian region.

Another useful consideration is related to the necessity of describing the low Q^2-tail of the D-function in the region where standard perturbative QCD, supplemented by quark and gluon condensate contributions, does not work. Among theoretical methods of the analysis of this low-energy tail are the methods of chiral perturbation theory and the approach based on the analytic model of the QCD running coupling constant (Reference: D.V. Shirkov and I.L. Solovtsov, *Phys. Rev. Lett.* **79** (1997) 1209; D.V. Shirkov and I.L. Solovtsov, hep-ph/9906495).

*) It should be emphasized that at high energies the perturbative QCD prediction starts to agree with the experimentally motivated behaviour of the Adler D-function only after inclusion of the mass dependence of the 3-loop order α_s^2-term.

*) Further application of the obtained results allowed to estimate the additional uncertainties of the hadronic vacuum polarization contribution to the running of fine coupling constant from low energies to t he scale, defined by the mass of the Z-boson (F. Jegerlehner, in "Radiative corrections: Applications of quantum field theory to phenomenology", hep-ph/9901386; A.D. Martin, J. Outhwaite and M.G. Ryskin, *J. Phys.* **G26** (2000) 600.) The results were also applied in the process of measurements of the running of the fine coupling constant in the Euclidean region using the Bhabha scattering data, recorded at LEP (Reference: L3 Collaboration, M. Acciarri et al., *Phys. Lett.* **B476** (2000) 40.)

*) It can be of interest to include into the performed analysis the new low-energy e^+e^--data, obtained at Novosibirsk and to refine the uncertainties of the hadronic contribution to the shift of the fine structure constant.