Two-loop QCD corrections to $q\bar{q} \to q'\bar{q}'$

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1 Notation

For calculational convenience, we treat all particles as incoming so that

$$q(p_1) + \bar{q}(p_2) + q'(p_3) + \bar{q}'(p_4) \to 0, \tag{1}$$

where the light-like momentum assignments are in parentheses and satisfy

$$p_1^\mu + p_2^\mu + p_3^\mu + p_4^\mu = 0.$$

The amplitude $\mathcal{M}$ for process (1) can be expanded in terms of the strong coupling $g = g_s\mu^\epsilon$

$$\mathcal{M} = g^2\mathcal{M}_0 + g^4\mathcal{M}_1 + g^6\mathcal{M}_2 + \mathcal{O}(g^8), \tag{2}$$

where $\mathcal{M}_i$ represents the i-loop amplitude. We denote the squared amplitude summed over spins and colours by

$$\sum |\mathcal{M}|^2 = \mathcal{A}(s,t,u), \tag{3}$$

where the Mandelstam variables are given by

$$s = (p_1+p_2)^2, \qquad t = (p_2+p_3)^2, \qquad u = (p_1+p_3)^2. \tag{4}$$

The summed and squared amplitude has the following perturbative expansion

$$\mathcal{A}(s,t,u) = g^4\mathcal{A}^4(s,t,u) + g^6\mathcal{A}^6(s,t,u) + g^8\mathcal{A}^8(s,t,u) + \mathcal{O}(g^{10}). \tag{5}$$

In terms of the amplitudes

$$\begin{aligned}
\mathcal{A}^4(s,t,u) &= \sum |\mathcal{M}_0|^2 = 2(N^2-1)\left(\frac{t^2+u^2}{s^2} - \epsilon\right), &(6)\\
\mathcal{A}^6(s,t,u) &= \sum \left(\mathcal{M}_0\mathcal{M}_1^\dagger + \mathcal{M}_1\mathcal{M}_0^\dagger\right), &(7)\\
\mathcal{A}^8(s,t,u) &= \sum \left(|\mathcal{M}_1|^2 + \mathcal{M}_0\mathcal{M}_2^\dagger + \mathcal{M}_2\mathcal{M}_0^\dagger\right). &(8)
\end{aligned}$$

We will concentrate on the interference between the tree level and the two-loop diagrams: $\mathcal{M}_0\mathcal{M}_2^\dagger + \mathcal{M}_2\mathcal{M}_0^\dagger$.

2 Method

In general, tensor integrals are associated with scalar integrals in higher dimension and with higher powers of propagators. This connection can straightforwardly be achieved using the Schwinger-parameter form of the integral. Systematic application of the integration-by-parts (IBP) identities and Lorentz invariance identities is sufficient to reduce these higher-dimension, higher-power integrals to master integrals in $D = 4 - 2\epsilon$.

Using these identities, we have constructed `MAPLE`, `MAXIMA` and `FORM` programs to rewrite two-loop tensor integrals for massless $2 \to 2$ scattering directly in terms of the basis set of master integrals.

The general procedure for computing the amplitudes is therefore as follows. First the one- and two-loop Feynman diagrams are generated using `QGRAF`. We then project by tree level, perform the summation over colours and spins and trace over the Dirac matrices in D dimensions. It is then straightforward to identify the scalar and tensor integrals present and replace them with combinations of master integrals. The final result is a combination of master integrals in $D = 4 - 2\epsilon$ which can be substituted for their expansions in ϵ.

3 Result for the interference of two-loop with tree-level

We divide the expansion into two parts: one proportional to the Born amplitude $\mathcal{A}^4$ and one that depends on kinematic structures that do not occur at tree level. Some of these non-tree level structures occur in the next-to-leading order term $\mathcal{A}^6$ and are therefore associated with single and double poles in ϵ in $\mathcal{A}^8$. The structure of $\mathcal{A}^8$ is thus

$$g^8\left(\mathcal{M}_0\mathcal{M}_2^\dagger + \mathcal{M}_2\mathcal{M}_0^\dagger\right) = g^4\ G_\epsilon\ \left(\frac{\alpha_s}{2\pi}\right)^2 (4\pi)^{2\epsilon}\left[\mathcal{A}^4(s,t,u)\ \sum_{i=0}^{4}\frac{X_i}{\epsilon^i} + (N^2-1)\sum_{i=0}^{2}\frac{Y_i}{\epsilon^i}\right], \quad (9)$$

where

$$G_\epsilon = \frac{\Gamma(1+\epsilon)^2\Gamma(1-\epsilon)^4}{\Gamma(1-2\epsilon)^2}. \quad (10)$$

The coefficients X_i and Y_i may be further subdivided by grouping terms according to the power of the number of colours N and the number of light quarks N_F

$$Z_i = N^2 A_i^Z + B_i^Z + \frac{1}{N^2}C_i^Z + N\,N_F D_i^Z + \frac{N_F}{N}E_i^Z + N_F^2 F_i^Z \quad \text{with} \quad Z_i = \{X_i, Y_i\}. \quad (11)$$

For convenience, we also introduce the following definitions

$$x = \frac{-t}{s}, \qquad y = \frac{-u}{s}, \qquad L_x = \log\left(\frac{-t}{s}\right), \qquad L_y = \log\left(\frac{-u}{s}\right), \qquad L_s = \log\left(\frac{s}{\mu^2}\right). \quad (12)$$

The leading and next-to-leading poles in ϵ have the simple structure

$$
\begin{aligned}
X_4 &= \frac{(N^2-1)^2}{N^2}, && (13)\\
X_3 &= (N^2-1)\left[-\frac{19}{12} - 2\,L_s - 2\,L_x + \frac{2\,L_s + 4\,L_x - 3 - 4\,L_y}{N^2} + \frac{5}{6}\frac{N_F}{N}\right]. && (14)
\end{aligned}
$$

Double poles:

$$
\begin{aligned}
A_2^X &= 2\,L_s^2 + \left(4\,L_x + \frac{19}{6}\right) L_s + \frac{1}{12}\,\pi^2 + L_x^2 + 2\,L_x - \frac{31}{12}\\
B_2^X &= -4\,L_s^2 + \left(8\,L_y - 12\,L_x + \frac{17}{6}\right) L_s + L_y^2 + \left(4 + 2\,L_x\right) L_y - \frac{17}{18} - \frac{1}{2}\,L_x\\
&\quad + \frac{11}{12}\,\pi^2 - 3\,L_x^2\\
C_2^X &= 2\,L_s^2 + \left(-6 + 8\,L_x - 8\,L_y\right) L_s + 3\,L_y^2 + \left(7 - 6\,L_x\right) L_y + 3\,L_x^2 - 7\,L_x - 2\,\pi^2 + \frac{41}{4}\\
D_2^X &= \frac{4}{9} - L_x - \frac{5}{3}\,L_s\\
E_2^X &= 2\,L_x + \frac{5}{3}\,L_s - \frac{26}{9} - 2\,L_y\\
F_2^X &= \frac{2}{9}\\
A_2^Y &= \left(-L_x^2 + L_x\right)\frac{t^2-u^2}{s^2} - 2\,\frac{ut}{s^2}\,L_x\\
B_2^Y &= \left(-2\,L_y + 3\,L_x^2 + 2\,L_y^2 - 3\,L_x\right)\frac{t^2-u^2}{s^2} + \left(6\,L_x - 4\,L_y\right)\frac{ut}{s^2}\\
C_2^Y &= \left(-2\,L_y^2 + 2\,L_x - 2\,L_x^2 + 2\,L_y\right)\frac{t^2-u^2}{s^2} + \left(-4\,L_x + 4\,L_y\right)\frac{ut}{s^2}\\
D_2^Y &= E_2^Y = F_2^Y = 0
\end{aligned}
$$

Single poles:

$$
\begin{aligned}
A_1^X &= -\frac{4}{3}\,L_s^3 + \left(-\frac{19}{6} - 4\,L_x\right) L_s^2 + \left(\frac{31}{6} - \frac{1}{6}\,\pi^2 - 2\,L_x^2 - 4\,L_x\right) L_s + \frac{1}{3}\,L_x^3 + \frac{1}{2}\,L_x^2\\
&\quad + \left(\frac{5}{6}\,\pi^2 + \frac{23}{4}\right) L_x - \frac{707}{432} + \frac{19}{36}\,\pi^2 - 2\,S_{12}(y) + \frac{7}{2}\,\zeta_3\\
B_1^X &= \frac{8}{3}\,L_s^3 + \left(12\,L_x - 8\,L_y - \frac{17}{6}\right) L_s^2 + \left(-2\,L_y^2 + \left(-4\,L_x - 8\right) L_y + 6\,L_x^2\right.\\
&\quad \left. + L_x + \frac{17}{9} - \frac{11}{6}\,\pi^2\right) L_s + \frac{2}{3}\,L_y^3 + \frac{1}{2}\,L_y^2 + \left(\frac{23}{2} + \frac{5}{3}\,\pi^2 + L_x\right) L_y - L_x^3 - \frac{3}{2}\,L_x^2\\
&\quad + \left(-\frac{9}{4} - \frac{17}{3}\,\pi^2\right) L_x + \frac{1219}{108} - \frac{1}{2}\,\zeta_3 - \frac{47}{18}\,\pi^2 + 6\,S_{12}(y) - 4\,S_{12}(x)
\end{aligned}
$$

$$
\begin{aligned}
C_1^X &= -\frac{4}{3}\,L_s^3+\Big(-8\,L_x+8\,L_y+6\Big)\,L_s^2\\
&+\Big(-6\,L_y^2+\Big(12\,L_x-14\Big)\,L_y-6\,L_x^2+14\,L_x-\frac{41}{2}+4\,\pi^2\Big)\,L_s-\frac{2}{3}\,L_y^3+\frac{3}{2}\,L_y^2\\
&+\Big(\frac{37}{2}-8\,\pi^2-3\,L_x\Big)\,L_y+\frac{2}{3}\,L_x^3+\frac{3}{2}\,L_x^2+\Big(-\frac{37}{2}+8\,\pi^2\Big)\,L_x+\frac{445}{16}\\
&-3\,\zeta_3-\frac{23}{4}\,\pi^2-4\,S_{12}(y)+4\,S_{12}(x)\\
D_1^X &= \frac{5}{3}\,L_s^2+\Big(2\,L_x-\frac{8}{9}\Big)\,L_s-2\,L_x-\frac{16}{9}-\frac{5}{18}\,\pi^2\\
E_1^X &= -\frac{5}{3}\,L_s^2+\Big(-4\,L_x+\frac{52}{9}+4\,L_y\Big)\,L_s+\frac{29}{18}\,\pi^2-4\,L_y-\frac{509}{54}+4\,L_x\\
F_1^X &= -\frac{4}{9}\,L_s+\frac{20}{27}\\
A_1^Y &= \Big(\Big(2\,L_x^2-2\,L_x\Big)\,L_s+\frac{5}{3}\,L_x^3+\frac{2}{3}\,L_x^2+\Big(\pi^2-\frac{7}{6}\Big)\,L_x-\frac{1}{2}\,\pi^2-2\,S_{12}(y)\Big)\,\frac{t^2-u^2}{s^2}\\
&+\Big(\pi^2+4\,L_s\,L_x+\frac{7}{3}\,L_x+2\,L_x^2\Big)\,\frac{ut}{s^2}\\
B_1^Y &= \Big(\Big(-4\,L_y^2+4\,L_y+6\,L_x-6\,L_x^2\Big)\,L_s-\frac{7}{3}\,L_y^3+\Big(-L_x-\frac{7}{3}\Big)\,L_y^2+\Big(L_x^2-2\,\pi^2+\frac{7}{3}\Big)\,L_y\\
&-5\,L_x^3+\frac{5}{3}\,L_x^2+\Big(-5\,\pi^2-\frac{1}{6}\Big)\,L_x+\frac{7}{2}\,\pi^2+6\,S_{12}(y)+4\,S_{12}(x)\Big)\,\frac{t^2-u^2}{s^2}\\
&+\Big(\Big(8\,L_y-12\,L_x\Big)\,L_s+2\,L_y^2+\Big(4\,L_x+\frac{14}{3}\Big)\,L_y-3\,\pi^2-6\,L_x^2+\frac{1}{3}\,L_x\Big)\,\frac{ut}{s^2}\\
C_1^Y &= \Big(\Big(4\,L_y^2+4\,L_x^2-4\,L_x-4\,L_y\Big)\,L_s-\frac{5}{3}\,L_y^3+\Big(-1+3\,L_x\Big)\,L_y^2+\Big(5-3\,L_x^2+6\,\pi^2\Big)\,L_y\\
&+\frac{13}{3}\,L_x^3-7\,L_x^2+\Big(5+6\,\pi^2\Big)\,L_x-6\,\pi^2-4\,S_{12}(y)-4\,S_{12}(x)\Big)\,\frac{t^2-u^2}{s^2}\\
&+\Big(\Big(8\,L_x-8\,L_y\Big)\,L_s+6\,L_y^2+\Big(-12\,L_x+10\Big)\,L_y-10\,L_x+6\,L_x^2\Big)\,\frac{ut}{s^2}\\
D_1^Y &= \Big(-\frac{2}{3}\,L_x^2+\frac{2}{3}\,L_x\Big)\,\frac{t^2-u^2}{s^2}-\frac{4}{3}\,\frac{ut}{s^2}\,L_x\\
E_1^Y &= \Big(\frac{4}{3}\,L_y^2-\frac{4}{3}\,L_x+\frac{4}{3}\,L_x^2-\frac{4}{3}\,L_y\Big)\,\frac{t^2-u^2}{s^2}+\Big(\frac{8}{3}\,L_x-\frac{8}{3}\,L_y\Big)\,\frac{ut}{s^2}\\
F_1^Y &= 0
\end{aligned}
$$

For the finite part see the forthcoming paper by C. Anastasiou, E. W. N. Glover, C. Oleari and M. E. Tejeda-Yeomans.